Topical Worksheet: Solving Quadratic Equations

Solving Quadratic Equations by Completing the Square

Question 1

(a) Express $x^2 + 6x - 2$ in the form $(x + a)^2 + b$.

(b) Hence, solve the equation $x^2 + 6x - 2 = 0$, giving your answers correct to 2 decimal places.

Question 2

(a) Express $x^2 - 4x - 7$ in the form $(x - a)^2 + b$.

(b) Hence, solve the equation $x^2 - 4x - 7 = 0$.

Question 3

(a) By completing the square, express $-x^2 + 8x + 15$ in the form of $-(x + a)^2 + b$.

(b) Hence, solve $-x^2 + 8x + 15 = 0$.

Question 4

(a) Show that the equation $2x^2 - 5x - 20$ can be expressed in the form $a(x - b)^2 + c$.

(b) Hence, solve the equation $2x^2 - 5x - 20 = 0$, giving your answer correct to 2 decimal places.

Solving Quadratic Equations by Quadratic Formula

Question 5

Solve the equation $3x^2 - 7x = 1$, giving your answers correct to 2 decimal places.

Question 6

Solve $h(h - 3) = 5h + 18$. Correct your answer to 3 significant figures.

Question 7

Solve the equation $(2x + 1)(x - 3) = 5$, giving your answers to 2 decimal places.

Question 8

Solve $(2x - 7)^2 = 6x + 3$ and give your answer correct to 2 decimal places.

ANSWERS:

1. (a) $x^2 + 6x - 2$

$= (x)^2 + 2(x)(3) - 2$

$= (x)^2 + 2(x)(3) + (3)^2 - (3)^2 - 2$

$= (x + 3)^2 - 9 - 2$

$= (x + 3)^2 - 11$

$\therefore a = 3, b = 11$

1. (b) $x^2 + 6x - 2 = 0$

$(x + 3)^2 - 11 = 0$

$(x + 3)^2 = 11$

$(x + 3) = \sqrt{11}$ or $x + 3 = -\sqrt{11}$

$x = \sqrt{11} - 3$ $x = -\sqrt{11} - 3$

$= 0.31662$ $= -6.31662$

$= 0.32$ (to 2 d.p.) $= -6.32$ (2 d.p.)

2. (a) $x^2 - 4x - 7$

$= (x)^2 - 2(x)(2) - 7$

$= (x)^2 - 2(x)(2) + (2)^2 - (2)^2 - 7$

$= (x - 2)^2 - 4 - 11$

$(x - 2)^2 - 11$

$\therefore a = 2, b = -11$

2. (b) $x^2 - 4x - 7 = 0$

$(x - 2)^2 - 11 = 0$

$(x - 2)^2 = 11$

$x - 2 = \sqrt{11}$ or $x - 2 = -\sqrt{11}$

$x = \sqrt{11} + 2$ $x = -\sqrt{11} + 2$

$= 5.3166$ $= -1.3166$

$= 5.32$ (2 d.p.) $= -1.32$ (2 d.p.)

3. (a) $-x^2+8x+15$

$=-(x^2-8x-15)$

$=-[(x)^2-2(x)(4)-15]$

$=-[(x)^2-2(x)(4)+(4)^2-(4)^2-15]$

$=-[(x-4)^2-16-15]$

$=-[(x-4)^2-31]$

$=-(x-4)^2+31$

$\therefore a=-4, b=31$

3. (b) $-x^2+8x+15=0$

$-(x-4)^2+31=0$

$(x-4)^2=31$

$x-4=\sqrt{31}$ or $x-4=-\sqrt{31}$

$x=\sqrt{31}+4$ $x=-\sqrt{31}+4$

$=9.5677$ $=-1.5677$

$=9.57$ (2 d.p.) $=-1.57$

4. (a) $2x^2-5x-20$

$=2(x^2-\frac{5}{2}x-10)$

$=2\left[(x)^2-2(x)\left(\frac{5}{4}\right)-10\right]$

$=2\left[(x)^2-2(x)\left(\frac{5}{4}\right)+\left(\frac{5}{4}\right)^2-\left(\frac{5}{4}\right)^2-10\right]$

$=2\left[\left(x-\frac{5}{4}\right)^2-\frac{25}{10}-10\right]$

$=2\left[\left(x-\frac{5}{4}\right)^2-\frac{185}{16}\right]$

$=2\left(x-\frac{5}{4}\right)^2-\frac{185}{8}$

$\therefore b=\frac{5}{4}, c=-\frac{185}{8}$

4. (b) $2x^2 - 5x - 20 = 0$

$2\left(x - \frac{5}{4}\right)^2 - \frac{185}{8} = 0$

$\left(x - \frac{5}{4}\right)^2 = \frac{185}{16}$

$x - \frac{5}{4} = \sqrt{\frac{185}{16}}$ or $x - \frac{5}{4} = -\sqrt{\frac{185}{16}}$

$x = 4.65036$ $x = -2.15036$

$= 4.65$ $x = -2.15$

5. $3x^2 - 7x = 1$

$3x^2 - 7x - 1 = 0$

$a = 3, b = -7, c = -1$

$x = -\frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-1)}}{2(3)}$

$x = \frac{7+\sqrt{61}}{6}$ or $x = \frac{7-\sqrt{61}}{6}$

$= 2.4683$ $x = -0.1350$

$= 2.47$ $x = -0.14$

6. $h(h - 3) = 5h + 18$

$h^2 - 3h = 5h + 18$

$h^2 - 3h - 5h - 18 = 0$

$h^2 - 8h - 18 = 0$

$a = 1, b = -8, c = -18$

$h = -\frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-18)}}{2(1)}$

$= \frac{8 \pm \sqrt{136}}{2}$

$= \frac{8+\sqrt{136}}{2}$ or $h = \frac{8-\sqrt{136}}{2}$

$= 9.8309$ $h = -1.8309$

$= 9.83$ $h = -1.83$

7. $(2x+1)(x-3)=5$

$2x(x-3)+1(x-3)=5$

$2x^2-6x+x-3-5=0$

$2x^2-5x-8=0$

$a=2, b=-5, c=-8$

$x=-\dfrac{-(-5)\pm\sqrt{(-5)^2-4(2)(-8)}}{2(2)}$

$=\dfrac{5\pm\sqrt{89}}{4}$

$=\frac{5\pm\sqrt{89}}{4}$ or $x=\frac{5-\sqrt{89}}{4}$

$=3.60849$ $x=-1.1084$

$=3.61$ $x=-1.11$

8. $(2x-7)^2=6x+3$

$(2x)^2-2(2x)(7)+(7)^2=6x+3$

$4x^2-28x+49=6x+3$

$4x^2-28x-6x+49-3=0$

$4x^2-34x+46=0$

$2x^2-17x+23=0$

$a=2, b=-17, c=23$

$x=\dfrac{-(-17)\pm\sqrt{(-17)^2-4(2)(23)}}{2(2)}$

$x=\dfrac{17\pm\sqrt{105}}{4}$

$x=\frac{17+\sqrt{105}}{4}$ or $x=\frac{17-\sqrt{105}}{4}$

$=6.8117$ $x=-1.6882$

$=6.81$ $x=-1.69$