Topical Worksheet: Functions and Graphs

Secondary 3 Mathematics

BASIC

1. For each of the following,
   (i) find the line of symmetry and the turning point,
   (ii) sketch the graph of the function.

   (a) $y = x^2 + 5$
   (b) $y = (x-1)^2 + 2$
   (c) $y = (x+4)^2 - 3$
   (d) $y = (x-3)^2 - 4$
   (e) $y = -x^2 + 4$
   (f) $y = -(x+2)^2 - 1$
   (g) $y = -(x+1)^2 + 6$
   (h) $y = -(x-3)^2 + 5$

2. For each of the following,
   (i) find the x-intercepts and the y-intercept
   (ii) state the line of symmetry and find the turning point,
   (ii) sketch the graph of the function

   (a) $y = (x+2)(x-2)$
   (b) $y = (x-1)(x-5)$
   (c) $y = (x+2)(x+5)$
   (d) $y = x(x+3)$
   (e) $y = -(x+6)(x-4)$
   (f) $y = -(x-1)(x-5)$
   (g) $y = -(x+5)(x+6)$
   (h) $y = -x(x-6)$

3. Draw the graphs of the following functions for $-4 \leq x \leq 4$.
   (a) $y = \frac{1}{2}x$
   (b) $y = -4x$
   (c) $y = \frac{3}{4}x^2$
   (d) $y = -4x^2$
   (e) $y = \frac{3}{2}x^3$
   (f) $-4x^3$

4. Draw the graphs of the following functions for $-4 \leq x \leq 4$.
   (a) $y = \frac{2}{x}$
   (b) $y = \frac{16}{x}$
   (c) $y = -\frac{3}{x}$
   (d) $y = -\frac{12}{x}$
   (e) $y = \frac{2}{x^2}$
   (f) $y = \frac{8}{x^2}$
   (g) $y = -\frac{4}{x^2}$
   (h) $y = -\frac{5}{x^2}$

5. Draw the graphs of the following functions for $-4 \leq x \leq 4$.
   (a) $y = 3x + 5$ (b) $y = x^2 - 3$
   (c) $y = x^3 - 4$ (d) $y = -x^3 + 2$
   (e) $y = \frac{1}{x} - 6$ (f) $y = -\frac{1}{x} + 1$
   (g) $y = \frac{1}{x^2} + 4$ (h) $y = -\frac{1}{x^2} - 2$

6. Draw the graphs of the following functions for $-4 \leq x \leq 4$.
   (a) $y = -3 + 2x^3$ (b) $y = x^3 + 3x$
   (c) $y = x^2 - x^3$ (d) $y = x^3 + x^2$
   (e) $y = x - \frac{2}{x}$ (f) $y = \frac{1}{x} + x^2$
   (g) $y = \frac{1}{x^2} + 2x$ (h) $y = x^3 - \frac{1}{x^2}$

7. Draw the graphs of the following functions for $-2 \leq x \leq 3$.
   (a) $y = 6^x$ (b) $y = -3^x$
   (c) $y = -5^x$ (d) $y = 2^{2x}$

8. Suppose n is a positive integer. Which of the above figures is possibly the graph of

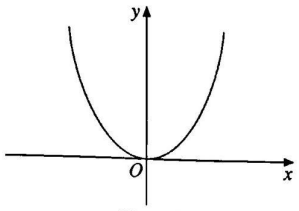


Figure I

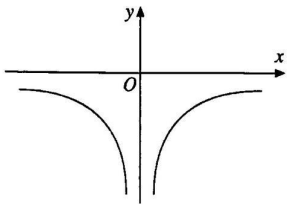


Figure II

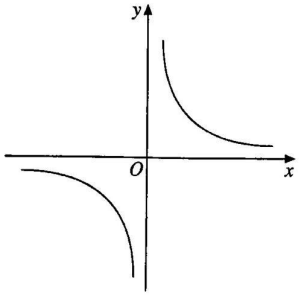


Figure III

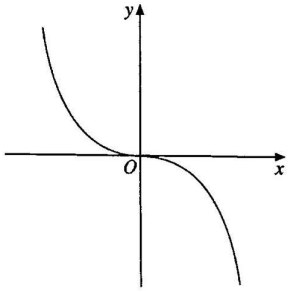


Figure IV

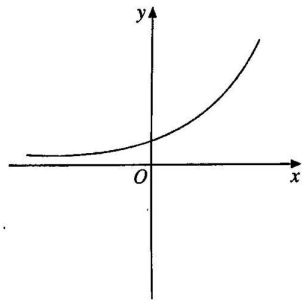


Figure V

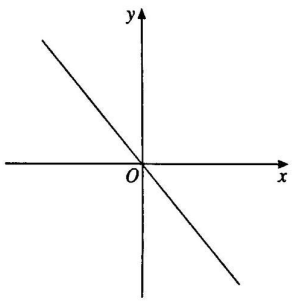
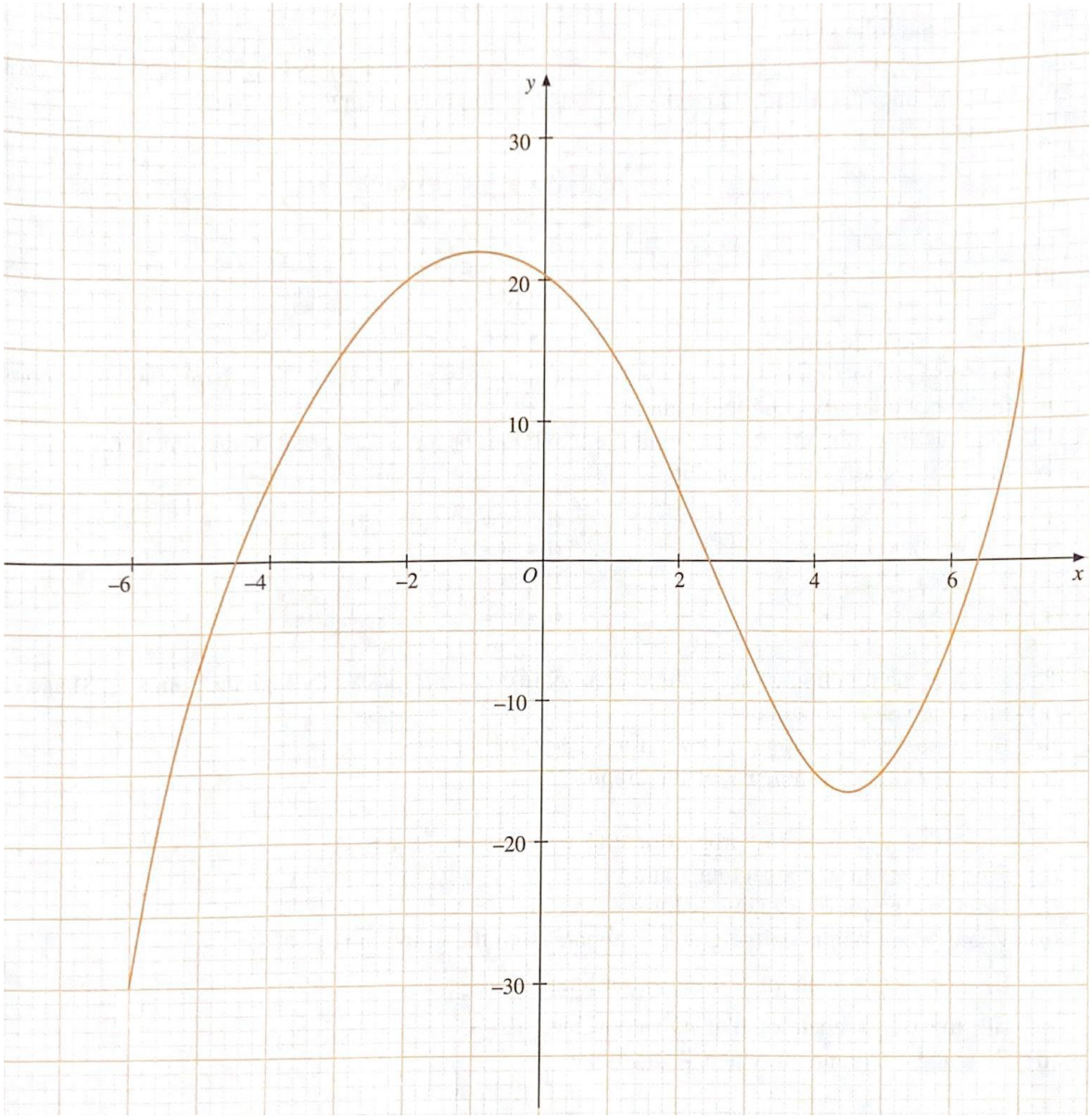


Figure VI

(a) $y = -nx$?
(b) $y = nx^2$?
(c) $y = -nx^3$?
(d) $y = \frac{n}{x}$?
(e) $y = -\frac{n}{x^2}$
(f) $y = n^x$?

9. The graph of a curve is shown in the diagram below.



By drawing a tangent, estimate the gradient of the curve at the point
(a) $A\ (-5, -7)$,
(b) $B\ (-2{,}20)$,
(c) $C\ (1{,}15)$,
(d) $D\ (5, -15)$.

10. (a) Draw the graph of the curve $y = (x + 2)(x - 4)$ for $-4 \leq x \leq 6$.
    (b) By drawing a tangent, estimate the gradient of the curve at the point when
    i. $x = -1$
    ii. $x = 4$

INTERMEDIATE

11. For each of the following,
    (i) express the function in the form $y = (x - h)^2 + k$, where h and k are constants.

    (ii) state the line of symmetry and the maximum or minimum point of the function,
    (iii) sketch the graph of the function.

    (a) $y = x^2 + 4x + 5$
    (b) $y = x^2 + 6x - 8$
    (c) $y = x^2 - 3x + 3$
    (d) $y = x^2 - 4x - 9$
    (e) $y = -x^2 + x + 4$
    (f) $y = -x^2 + 2x - 6$
    (g) $y = -x^2 - 5x + 7$
    (h) $y = -x^2 - 8x - 10$

12. For each of the following,
    (i) express the function in the form $y = (x - a)(x - b)$ or $y = -(x - a)(x - b)$, where a and b are constants,
    (ii) find the x-intercepts and the y-intercept,
    (iii) state the line of symmetry and find the maximum or minimum point of the function,
    (iv) sketch the graph of the function.

    (a) $y = x^2 + 4x + 3$
    (b) $y = x^2 + 6x - 16$
    (c) $y = x^2 - 11x + 24$
    (d) $y = x^2 - 16$
    (e) $y = -x^2 + 2x + 3$
    (f) $y = -x^2 + 5x - 6$
    (g) $y = -x^2 - 8x + 20$
    (h) $y = -x^2 + 25$

13. The graph of the function $y = kx^r$, where k and r are constants, passes through the points (1,3) and (2,12).
    (a) Find the values of k and r.
    (b) Hence, draw the graph of $y = kx^r$ for $-2 \leq x \leq 2$.
    (c) State the line of symmetry of the graph.

14. It is given that y is inversely proportional to x. When $x = \frac{1}{2}$, $y = -8$.
    (a) Find the equation connecting x and y.
    (b) Draw the graph of y against x for $-4 \leq x \leq 4$.
    (c) V and W are points on the graph. Estimate from the graph in (b),

i. the y-coordinate of V if its x-coordinate is $-1\frac{1}{2}$.
ii. the x-coordinate of W if its y-coordinate is -3.

(d) State the quadrants in which V and W lie.

15. (a) Draw the graph of $y = x^2 + \frac{2}{x}$ for $0 < x \leq 4$.
(b) Find the turning point of the graph.
(c) Use your graph to solve the equation $x^2 + \frac{2}{x} = 6$

16. (a) Draw the graph of $y = x^3 - 4x^2 + 4$ for $-2 \leq x \leq 5$.
(b) Use your graph to solve the equation
i. $x^3 - 4x^2 + 4 = 0$,
ii. $x^3 - 4x^2 + 8 = 0$,
iii. $x^3 - 4x^2 + 4 = x$.

17. (a) Draw the graph of $y = \frac{1}{4}(5^x)$ for $-1 \leq x \leq 3$.
(b) Find, from the graph,
i. the value of y when $x = 2.5$,
ii. the value of x when $y = 25$.

18. The diagram shows a sketch of the graph of $y = ka^x$, where a and k are constants. The graph passes through the points $(0,2)$ and $(3,16)$.
(a) Find the values of k and a.
(b) Hence, use the answers in (a) to simplify ka^x as a single power of a.

19. (a) Draw the graph of the curve $y = x^3 + 3x^2 - 9x$ for $-4 \leq x \leq 4$.
(b) Estimate the maximum and minimum points of the graph.
(c) Hence, find the interval of values of x for which the gradient of the curve is non-positive for $-4 \leq x \leq 4$.

20. (a) Draw the graphs of the functions $y = 6x$ and $y = x^2$ for $0 \leq x \leq 8$.
(b) Use your graphs to solve $6x = x^2$.
(c) (i) Locate graphically a point on the graph of $y = x^2$ for which the gradient is 6.
(ii) Write down the coordinates of this point.

ANSWERS

1. (a)(i) $x = 0, (0,5)$ (b)(i) $x = 1, (1,2)$
(c)(i) $x = -4, (-4,-3)$ (d)(i) $x = 3, (3,-4)$
(e)(i) $x = 0, (0,4)$ (f)(i) $x = -2, (-2,-1)$
(g)(i) $x = -1, (-1,6)$ (h)(i) $x = 3, (3,5)$

2. (a)(i) x-intercepts: $2, -2$, y-intercept: -4 (ii) $x = 0, (0,-4)$
(b)(i) x-intercepts: $1, 5$, y-intercept: 5 (ii) $x = 3, (3,-4)$
(c)(i) x-intercepts: $-2, -5$, y-intercept: 10
(c)(ii) $x = -1.5, (-3\ .5, -2.25)$
(d)(i) x-intercepts: $0, -3$, y-intercept: 0
(d)(ii) $x = -1.5, (-1.5, -2.25)$
(e)(i) x-intercepts: $-6{,}4$, y-intercept: 24 (ii) $x = -1, (-1{,}25)$
(f)(i) x-intercepts: $1{,}5$, y-intercept: -5 (ii) $x = 3, (3{,}4)$
(g)(i) x-intercepts: $-5, -6$, y-intercept: -30
(g)(ii) $x = -5.5, (-5.5, 0.25)$
(h)(i) x-intercepts: $0{,}6$, y-intercept: 0 (ii) $x = 3, (3{,}9)$

8. (a) Figure VI (b) Figure I (c) Figure IV
(d) Figure III (e) Figure II (f) Figure V

9. (a) 17 (b) 3.75 (c) $-7\frac{1}{3}$ (d) $6\frac{1}{3}$

10. (b)(i) -4 (ii) 6

11. (a)(i) $y = (x+2)^2 + 1$ (ii) $x = 2, (-2{,}1)$
(b)(i) $y = (x+3)^2 - 17$ (ii) $x = -3, (-3, -17)$
(c)(i) $y = \left(x - \frac{3}{2}\right)^2 + \frac{3}{4}$ (ii) $x = \frac{3}{2}, \left(\frac{3}{2}, \frac{3}{4}\right)$
(d)(i) $y = (x-2)^2 - 13$ (ii) $x = 2, (2, -13)$
(e)(i) $y = -\left(x - \frac{1}{2}\right)^2 + 4\frac{1}{4}$ (ii) $x = \frac{1}{2}, (\frac{1}{2}, 4\frac{1}{4})$
(f)(i) $y = -(x-1)^2 - 5$ (ii) $x = 1, (1, -5)$
(g)(i) $y = -\left(x + \frac{5}{2}\right)^2 = 13\frac{1}{4}$ (ii) $x = -2\frac{1}{2}, (-2\frac{1}{2}, 13\frac{1}{4})$
(h)(i) $y = -(x+4)^2 + 6$ (ii) $x = -4, (-4{,}6)$

12. (a)(i) $y = (x+3)(x+1)$ (ii) x-intercepts: $-1{,}3$, y-intercept: 3
(iii) $x = -2, (-2, -1)$
(b)(i) $y = (x+8)(x-2)$ (ii) x-intercepts: $-8{,}2$, y-intercept: -16
(iii) $xx = -3, (-3, -25)$
(c)(i) $y = (x-3)(x-8)$ (ii) x-intercepts: $3{,}8$, y-intercept: 24
(iii) $x = 5.5, (5.5, -6.25)$
(d)(i) $y = (x+4)(x-4)$ (ii) x-intercepts: $4, -4$, y-intercept: -16
(iii) $x = 0, (0, -16)$
(e)(i) $y = -(x-3)(x+1)$ (ii) x-intercepts: $-1, 3$, y-intercept: 3
(iii) $x = 1, (1{,}4)$
(f)(i) $y = -(x-3)(x-2)$ (ii) x-intercepts: $2, 3$, y-intercept: -6
(iii) $x = 2.5, (2.5, 0.25)$
(g)(i) $y = -(x-2)(x+10)$ (ii) x-intercepts: $2, -10$, y-intercept: 20
(iii) $x = -4, (-4, 36)$
(h)(i) $y = -(x-5)(x+5)$ (ii) x-intercepts: $5, -5$, y-intercept: 25

(iii) $x = 0, (0, 25)$

13. (a) $k = 3, r = 2$ (c) $x = 0$
14. (a) $y = -\frac{4}{x}$ (c)(i) 2.7 (c)(ii) 1.3
    (d) V – 2nd quadrant, W – 4th quadrant
15. (b) (1,3) (c) $x = 0.35 \; or \; x = 2.25$
16. (b)(i) $x = -0.9, 1.15 \; or \; 3.7$ (ii) $x = -1.25, 2 \; or \; 3.1$
    (iii) $x = -1, 1 \; or \; 4$
17. (b)(i) 14 (ii) 2.9
18. (a) $k = 2, a = 2$ (b) 2^{x+1}
19. (b) maximum point: $(-3, 27)$ minimum point: $(1, -5)$
    (c) $-3 \leq x \leq 1$
20. (b) $x = 0 \; or \; x = 6$ (c)(ii) $(3, 9)$