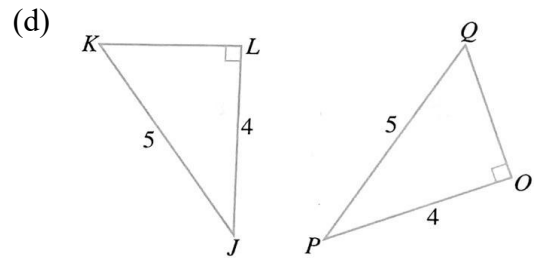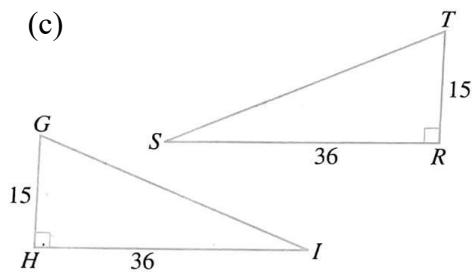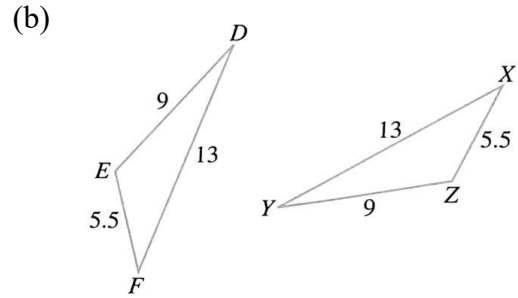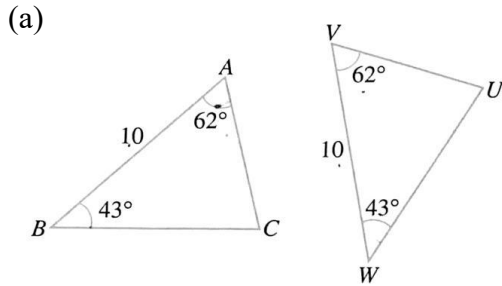# Topical Worksheet: Conditions of Congruence and Similarity
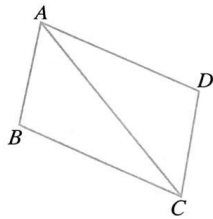## Secondary 3 Mathematics

**BASIC**

1. In each of the following, name the pair of congruent triangles and state the reason for the congruence.
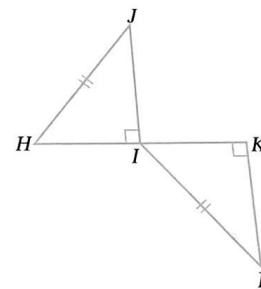
(a)



(b)



(c)



(d)



2. In each of the following, name the pair of congruent triangles and state the reason for the congruence.
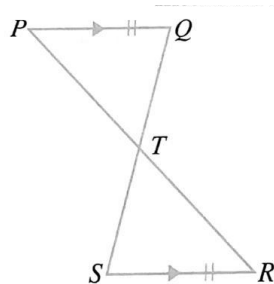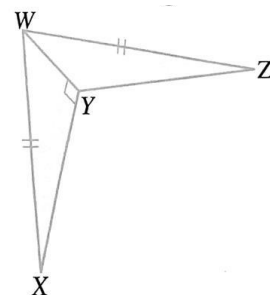
(a) $ABCD$ is a parallelogram.



(b) $I$ is the midpoint of $HK$.
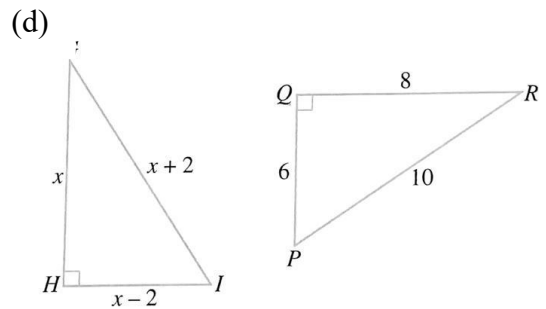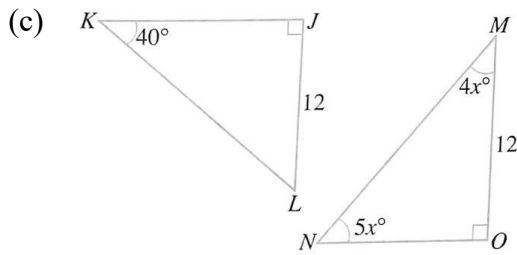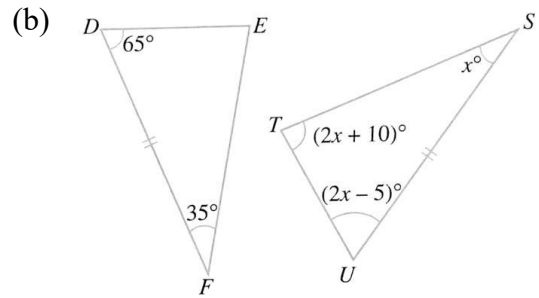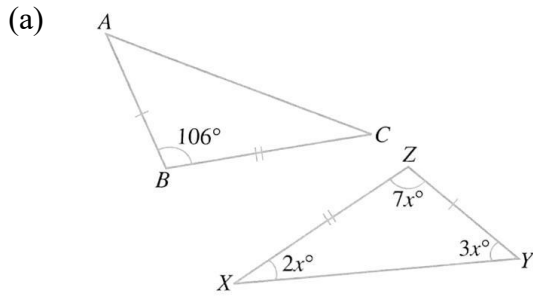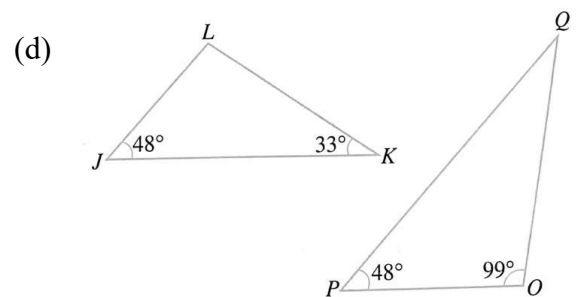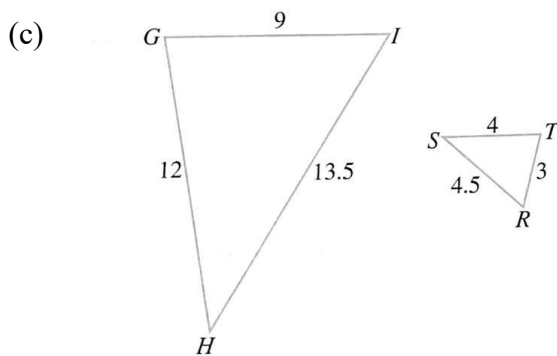


(c) $PR$ and $QS$ intersect at $T$.



(d) $WY$ is the angle bisector of $\angle XYZ$.

3.  Find the value of $x$ for each of the following. Hence, determine whether the triangles are congruent. If so, state the reason.

(a)



(b)



(c)



(d)



4.  In each of the following, name the pair of similar triangles and state the reason for the similarity.

(a)



(b)



(c)



(d)
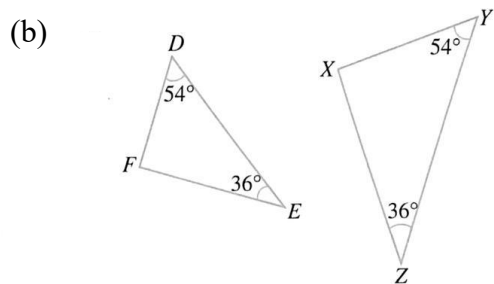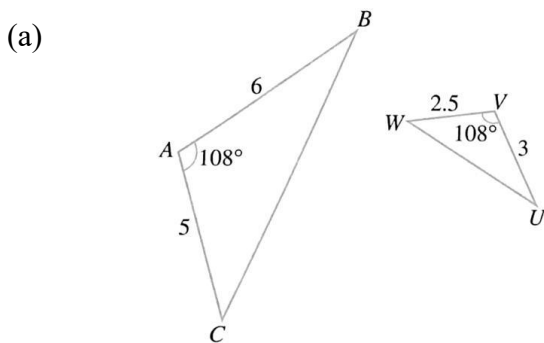
5. In each of the following, determine whether the triangles are similar. If they are, give the reason for the similarity.

(a)



(b)



(c)



(d)
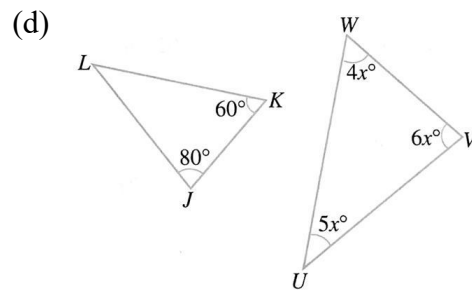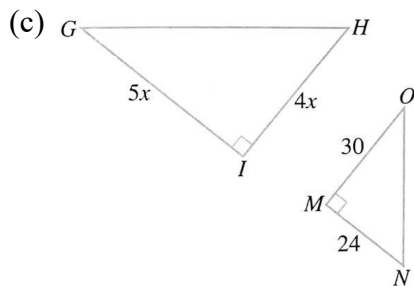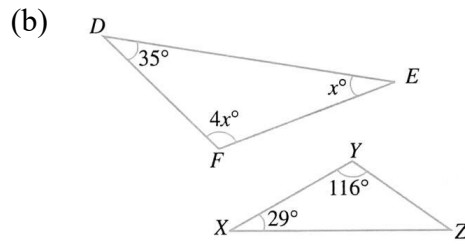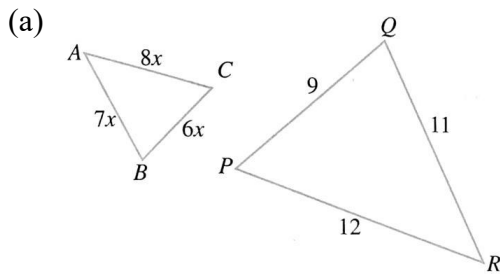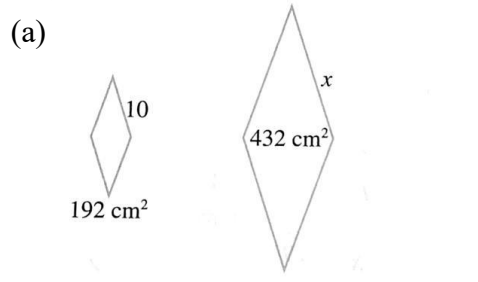


6. Find the unknown area $A_1$ and $A_2$ for each of the following pairs of similar figures. The units of length and area are cm and cm$^2$ respectively.

(a)



$A_1$  5   $A_2 = 126$   15

(b)



$A_1 = 22.5$   $A_2$

3   5

(c)



$w$   2.5$w$
$A_1 = 68$   $A_2$

(d)



$A_1$   $A_2 = 392$
3$x$   7$x$
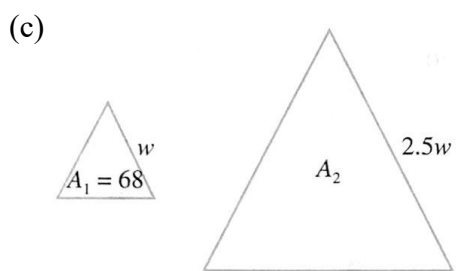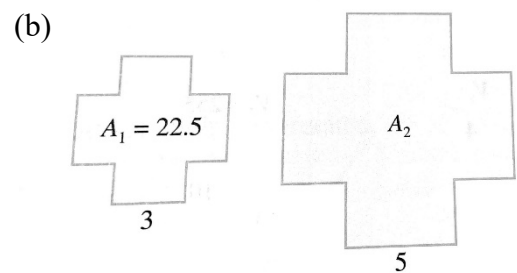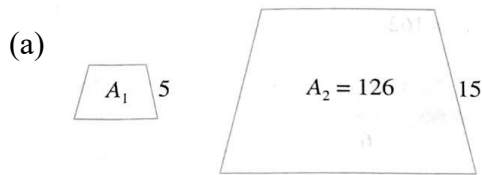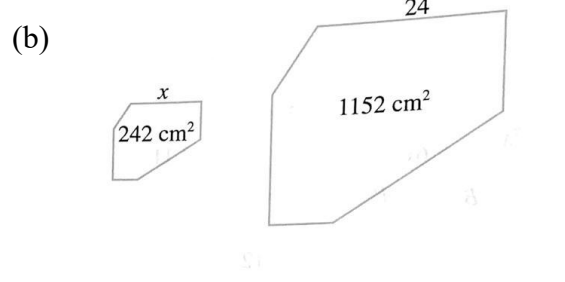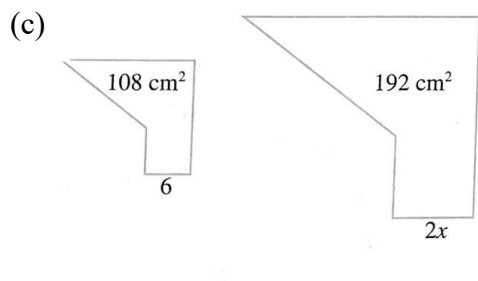
(e) The perimeters of the small and large figures are 3$y$ cm and 5$y$ cm respectively.



$A_1 = 396$   $A_2$
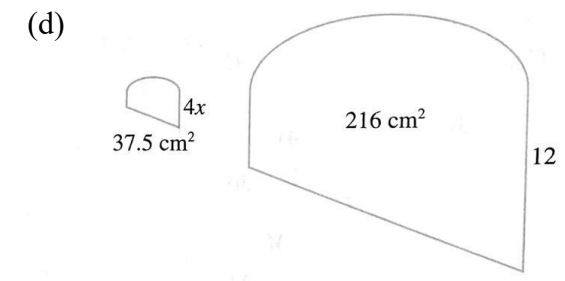
7. Find the value of $x$ for each of the following pairs of similar figures. The unit of length is $cm$.

(a)

10

432 cm²

$x$

192 cm²

(b)

24

$x$

242 cm²

1152 cm²

(c)

108 cm²

6

192 cm²

2x

(d)

4x

37.5 cm²

216 cm²

12
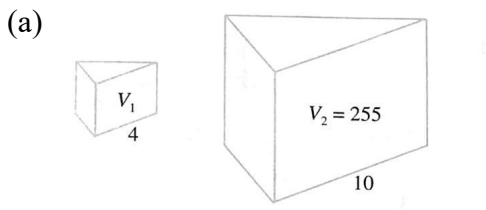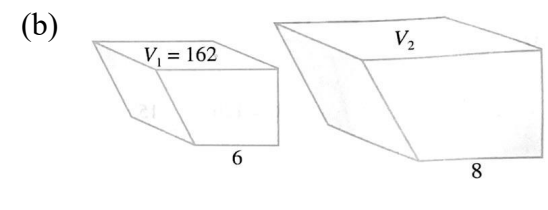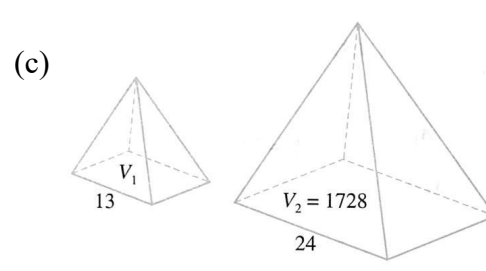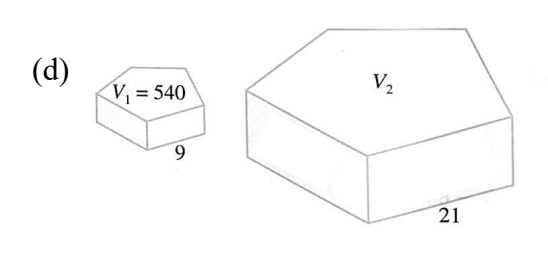
8. Find the unknown volume $V_1$ or $V_2$ for each of the following pairs of similar solids. The units of length and volume are cm and cm³ respectively.

(a)

$V_1$

4

$V_2 = 255$

10

(b)

$V_1 = 162$

6

$V_2$

8

(c)

$V_1$

13

$V_2 = 1728$

24

(d)

$V_1 = 540$

9

$V_2$

21

9. Find the unknown side $x$ for each of the following pairs of similar solids. The unit of length is cm.

(a)

40 cm³

1080 cm³

$x$

12

(b)

$x$

4

704 cm³

3773 cm³

(c)

24 cm³

375 cm³

2.5

$x$

(d)

5

1250 cm³

$x$

80 cm³
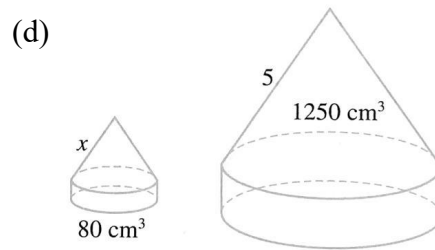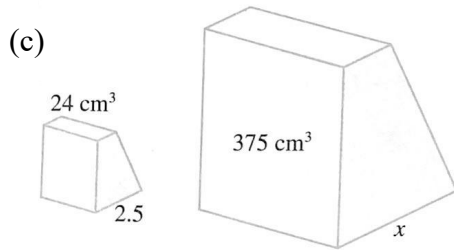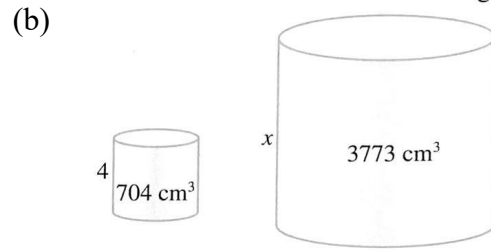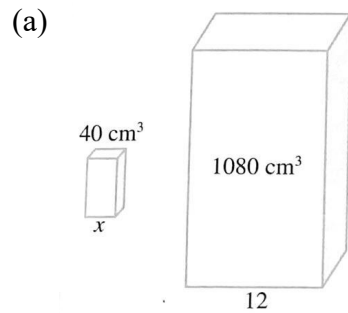
10. Find the ratio of the base areas of two similar solids if the ratio of their volumes is
   (a) $1:27$
   (b) $8:125$
   (c) $729:1331$
   (d) $512:343$
   (e) $3375:216$
   (f) $4096:1000$

11. In each of the following diagrams,
   (i)  name a pair of congruent triangles and state the reason for their congruence,
   (ii) find the value of $x$

(a)   $BD$ and $CE$ intersect at $A$.
      $CE = 27$ cm.

$B$

15

$A$

$x$

$D$

$E$

(b)

$T$

$11x°$

$S$

$3x°$

$4x°$

$U$

$V$

(c)

$J$

10

26

$I$

$H$

$x$

10

$F$

26

$G$
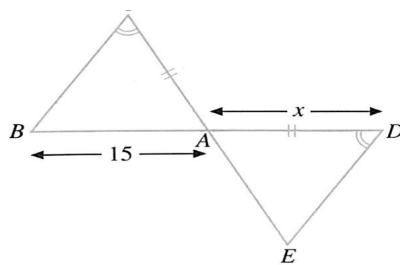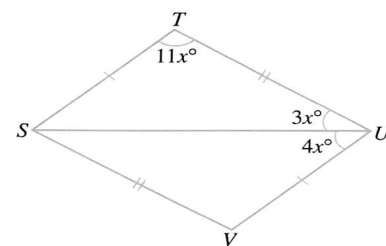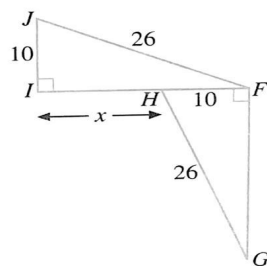
(d)   $NP$ and $MQ$ intersect at $O$.
      $PQR$ is a straight line.

$P$

$Q$

$R$

$115°$

$x°$

$O$

$M$

$48°$

$N$
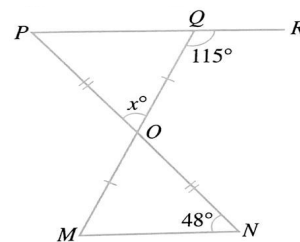
12. In the figure, $ABCD$ is a square and $\angle AFE = 45°$.
    (a) Show that $\triangle CED$ is a congruent to $\triangle CFB$.
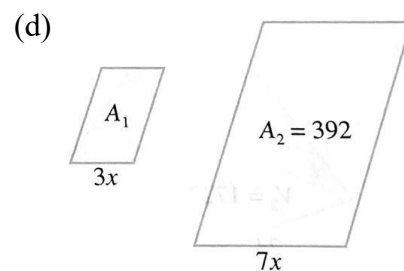    (b) Show that $\triangle CEF$ is a isosceles triangle.

45°
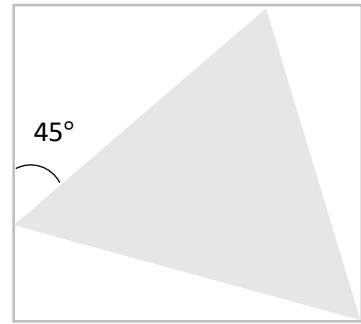
13. In each of the following, name the pair of similar triangles and state the reason for the similarity.

(a)  $D$ and $E$ are points on $AB$ and $AC$ respectively. $DE \;/\!/\; BC$.
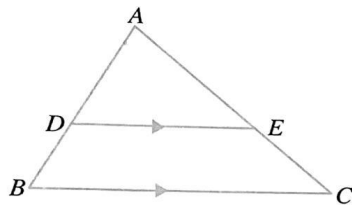
(b)  $J$ is a point on $HL$.
$HI\!:\!LK = IJ\!:\!KH = HJ\!:\!LH$.
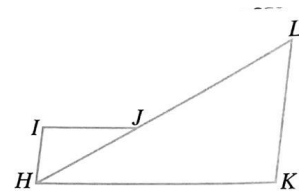
(c)  $WXYZ$ is a trapezium. Its diagonals intersect at $V$.
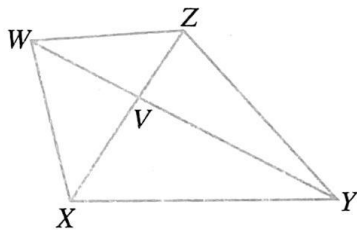
(d)  $S$ is a point on $PR$.
$PS \times QR = PR \times TS$.
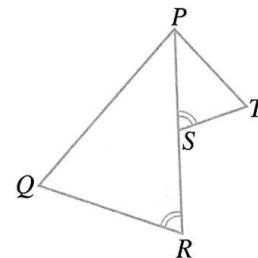
14. In each of the following diagrams, all measurements are in cm.
    (i) Name a pair of similar triangles and state the reason for their similarity.
    (ii) Find the value of $x$.

(a)  $I$ is a point on $FH$.

18, $x$, 24, 45

(b)  $C$ is a point on $BE$.

8.4, 4.2, 6, $x$, 135°
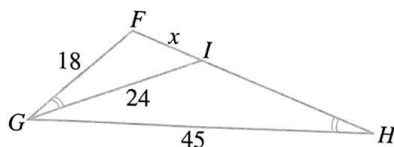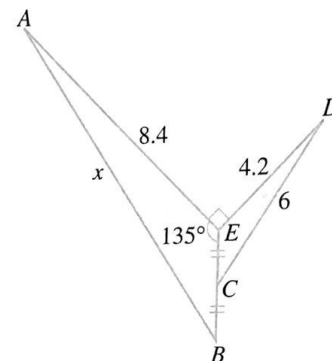
(c)   $S$ and $T$ are on $PQ$ and $PR$ respectively.
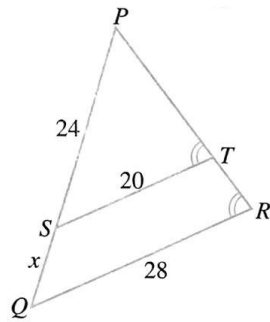


(d)   $JML$ is a straight line and $JL = 23\frac{1}{2}$ cm.
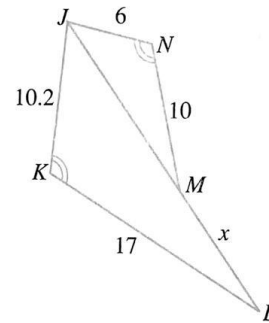


15. In each of the following diagrams, the lengths area in cm, find the values of $x$ and $y$.

(a)   $D$ and $E$ are on $BC$ and $AC$ respectively.
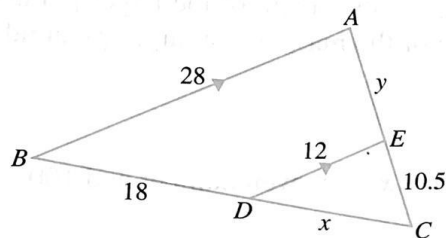


(b)   $FI$ and $GJ$ intersect at $H$ and $FI = 16\frac{1}{4}$ cm.
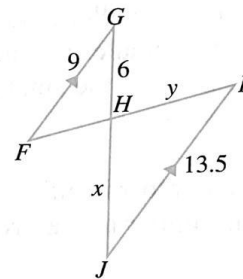


(c)   $MNO$, $OPR$ and $NPQ$ are straight lines.
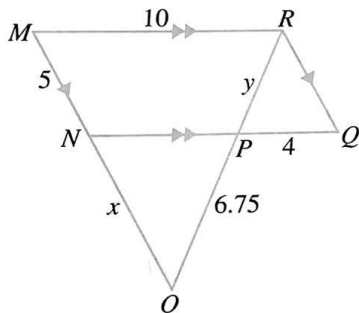


(d)   $WKZ$ and $STU$ are parallel lines. $WZ = 7$ cm.
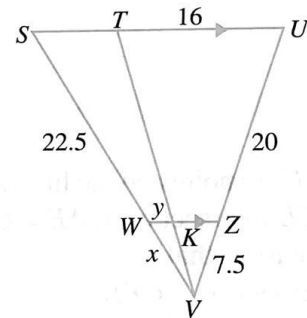


16. The lengths of the shortest sides of a pair of similar hexagons are 15 cm and 27 cm respectively.
   (a) Find the ratio of
           (i)  the perimeters of the two hexagons,
           (ii) the areas of the two hexagons.
   (b) Calculate the perimeter of the larger hexagon if the perimeter of the smaller hexagon is 143 cm.
   (c) Calculate the area of the smaller hexagon if the area of the larger hexagon is 2916 cm².

17. In the figure, $AB$ is parallel to $DE$ and $AD = 8$
    cm.  The areas of $\triangle CDE$ and quadrilateral $ABED$
    are 192 cm$^2$ and 108 cm$^2$ respectively.
    (a) Name a pair of similar triangles
        and state the reason for the similarity.
    (b) Find the length of $CD$.
    (c) Calculate the perpendicular distance from
        $B$ to $AC$.
    (d) Use your answer in (c) to calculate the
        perpendicular distance from $E$ to $AC$.
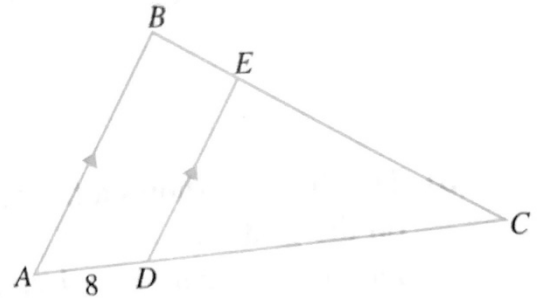
18. In the figure, $WXYZ$ is a parallelogram,
    $WY$ and $ZP$ intersect at $S$ such that $WS : WY = 3 : 5$.
    The length of $XP$ is 12 cm and the area of
    $\triangle YSZ$ is 162 cm$^2$.
    (a) Show that $\triangle WSZ$ is similar to $\triangle$
        YSP.
    (b) Find the length of $WZ$.
    (c) Find the area of
        (i)  $\triangle$ WSZ,
        (ii) $\triangle YSP$,
        (iii) $PSWX$.
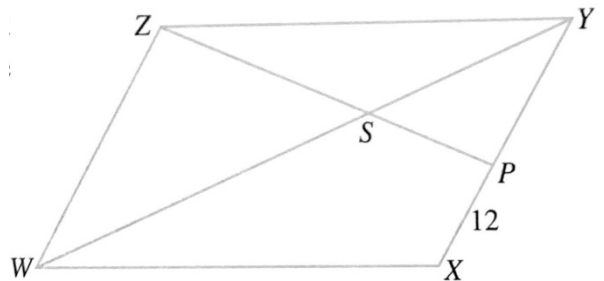
19. Two similar solids pyramids are made of the same material. The mass and base area
    of the larger pyramid are 4500 g and 1800 cm$^2$ respectively. The base area of the
    smaller pyramid is 648 cm$^2$.
    (a) Express the height of the smaller pyramid as a percentage of the height of the
        larger pyramid.
    (b) Express the mass of the smaller pyramid as percentage of the mass of the
        larger pyramid.
    (c) Hence, calculate the mass of the smaller pyramid.

20. A cubical jelly block is cut into $x$ identical cubical blocks where $x$ is an even number
    and $100 \leq x \leq 400$.
    (a) Find the value of $x$.
    (b) Suppose that the length of a side of the original jelly blocks is 15.6 cm. Find
        the total surface area of each small jelly block.

**ANSWERS**

1. (a) △ ABC ≡ △ VWU (ASA)           (b) △ DEF ≡ △ YZX (SSS)
   (c) △ GHI ≡ △ TRS ($SAS$)          (d) △ JKL ≡ △ PQO (RHS)

2. (a) △ ABC ≡ △ CDA (SSS)           (b) △ HIJ ≡ △ IKL (RHS)
   (c) △ PQT ≡ △ RST (ASA)           (d) △ WXY ≡ △ WZY (SAS)

3. (a) $x = 15$, not congruent         (b) $x = 35$, △ DEF ≡ △ UTS (ASA)
   (c) $x = 10$, not congruent         (d) $x = 8$, △ GHI ≡ △ RQP (SSS)

4. (a) △ ABC is similar to △ VUW. (Side-Angle-Side similarity)
   (b) △ DEF is similar to △ YZX. (Angle-Angle-Angle similarity)
   (c) △ GHI is similar to △ TSR. (Side-Side-Side similarity)
   (d) △ JKL is similar to △ PQO. (Angle-Angle-Angle similarity)

5. (a) not similar
   (b) △ DEF is similar to △ ZXY. (Angle-Angle-Angle similarity)
   (c) △ GHI is similar to △ ONM. (Side-Angle-Side similarity)
   (d) not similar

6. (a) $A_1 = 14\,\text{cm}^2$           (b) $A_2 = 62.5\,\text{cm}^2$           (c) $A_2 = 425\,\text{cm}^2$
   (d) $A_1 = 72\,\text{cm}^2$           (e) $A_2 = 1100\,\text{cm}^2$

7. (a) 15                 (b) 11                 (c) 4                 (d) 1.25

8. (a) $V_1 = 16.32\,\text{cm}^3$           (b) $V_2 = 384\,\text{cm}^3$
   (c) $V_1 = 274.625\,\text{cm}^3$         (d) $V_2 = 6860\,\text{cm}^3$

9. (a) 4                 (b) 7                 (c) 6.25                 (d) 2

10. (a) $1:9$                 (b) $4:25$                 (c) $81:121$                 (d) $64:49$
    (e) $25:4$                (f) $64:25$

11. (a)(i) △ ABC ≡ △ AED (ASA)           (ii) 12
    (b)(i) △ STU ≡ △ UVS (SSS)                (ii) 10
    (c)(i) △ FGH ≡ △ IFJ (RHS)                (ii) 14
    (d)(i) △ MCO ≡ △ QPO (SAS)                (II) 67

13. (a) △ ABC is similar to △ ADE. (Angle-Angle-Angle similarity)
    (b) △ HIJ is similar to △ LKH. (Side-Side-Side similarity)
    (c) △ VWZ is similar to △ VYX. (Angle-Angle-Angle similarity)
    (d) △ PQR is similar to △ PTS. (Side-Angle-Side similarity)

**14.** (a)(i) $\triangle$ FGI is similar to $\triangle$ FHG. (Angle-Angle-Angle similarity)        (ii) 9.6
    (b)(i) $\triangle$ ABE is similar to $\triangle$ DCE. (Side-Angle-Side similarity)         (ii) 12
    (c)(i) $\triangle$ PST is similar to $\triangle$ PQR. (Angle-Angle-Angle similarity)        (ii) 9.6
    (d)(i) $\triangle$ JKL is similar to $\triangle$ JNM. (Side-Angle-Side similarity)
        (ii) 9.61 or $9\frac{31}{51}$

**15.** (a) $x = 13.5, y = 14$                 (b) $x = 9, y = 9.75$
    (c) $x = 7.5, y = 4.5$               (d) $x = 8\frac{7}{16}, y = 2\frac{7}{11}$

**16.** (a)(i) $5:9$                 (ii) $25:81$
    (b) 257.4 cm              (c) 900 cm$^2$

17. (a) $\triangle$ ABC is similar to $\triangle$ DEC. (Angle-Angle-Angle similarity)
    (b) 32 cm             (c) 15 cm             (d) 12 cm

18. (b) 36 cm
    (c)(i) 243 cm$^2$             (ii) 108 cm$^2$           (iii) 297 cm$^2$

19. (a) 60%                (b) 21.6%             (c) 972 $g$

20. (a) 216                 (b) 40.56 cm$^2$