

## Topical Worksheet: Linear Inequalities

## Secondary 3 Mathematics

**BASIC**

1. Fill in each box with an inequality sign to make each of the following statements correct.

(a) If  $x > y$ , then  $x + 4$    $y + 4$ .

(b) If  $m \leq n$ , then  $3m$    $3n$ .

(c) If  $c < d$ , then  $-5c$    $-5d$ .

(d) If  $v \geq w$ , then  $\frac{v}{-2}$    $\frac{w}{-2}$ .

(e) If  $a > b$ , then  $2a - 7$    $2b - 7$ .

(f) If  $r \leq s$ , then  $-8r + 9$    $-8s + 9$ .

(g) If  $p = q + 6$ , then  $p$    $q$ .

(h) If  $g = h - 10$ , then  $g$    $h$ .

2. Solve each of the following inequalities and represent the solution on a number line.

(a)  $x + 5 > 12$

(b)  $2x - 9 < 21$

(c)  $3x + 10 \geq 16$

(d)  $8 - x < 15$

(e)  $17 - 3x \geq 26$

(f)  $4x + 13 < 33 - x$

(g)  $45 - 8x \leq 5x - 7$

(h)  $\frac{x}{2} + 7 > 5$

3. (a) Find the smallest integer value of  $x$  that satisfies each of the following inequalities.

(i)  $2x - 7 > 15$

(ii)  $5x + 9 > x$

(iii)  $6x - 13 \geq 4x - 15$

(iv)  $1 - 7x \leq 8 - 2x$

(b) Find the greatest integer value of  $x$  that satisfies each of the following inequalities.

(i)  $3x + 8 < 20$

(ii)  $10x - 25 \leq 6x$

(iii)  $x + 19 \leq 2 - x$

(iv)  $4(9 - x) \geq 3(2 + x)$

4. Solve each of the following pairs of simultaneous inequalities. Represent the solution on a number line.

(a)  $x + 1 > 15$  and  $2x - 3 < 35$

(b)  $4x - 12 < x$  and  $5x + 26 > 11$

(c)  $15x - 14 > 8x$  and  $36 - 4x > 9 - x$

(d)  $3(1 - 5x) < 2(5x + 14)$  and  $5(6 - 3x) \leq 3(2x - 11)$

(e)  $28 - x \leq 3x + 4$  and  $\frac{1}{2}x + 14 < 2 + x$

(f)  $x \geq \frac{5x+18}{11}$  and  $\frac{2}{3}x - 5 < \frac{x-5}{4}$

(g)  $2x > \frac{7x-9}{3}$  and  $\frac{2x-7}{3} \leq 4x + 1$

(h)  $2x - 3 \leq \frac{4x+5}{3}$  and  $\frac{5(3-4x)}{7} \leq \frac{7-3x}{2}$

5. Find the smallest and greatest integer values of  $x$  that satisfy each of the following pairs of simultaneous inequalities.

(a)  $x + 12 \geq 17$  and  $3x - 8 < 25$

(b)  $30 - 2x > 14$  and  $\frac{1}{2}x + 1 \geq 2\frac{1}{2}$

(c)  $2x \geq \frac{x-30}{3}$  and  $x - 1 < \frac{2x+21}{5}$

(d)  $30 - 2x > 14$  and  $\frac{1}{2}x + 1 \leq 7\frac{1}{2}$

(e)  $\frac{x-11}{3} \leq \frac{x+2}{4}$  and  $2x - 7 \geq \frac{9-x}{2}$

(f)  $2x + 1 > \frac{3-x}{5}$  and  $\frac{2(x-3)}{3} \leq \frac{8-x}{4}$

(g)  $\frac{x}{3} - 20 \leq \frac{x}{6}$  and  $\frac{3(x-5)}{2} \geq \frac{5(x+5)}{9}$

6. To pass a culinary course with distinction, a candidate must score at least 75 marks in the culinary examination. The examination consists of a practical component and a theory component and the maximum scores of the 2 components are 60 marks and 40 marks respectively.

(a) Mrs Lee scores 51 marks in the practical component. What should she score in the practical component to obtain a distinction?

(b) Sisca scores 30 marks in the theory component. What should she score in the practical component to obtain a distinction?

7. Wen Hui buys 10 apples and  $x$  oranges from a supermarket. Suppose that the costs of an apple and an orange are \$0.60 and \$0.55 respectively.

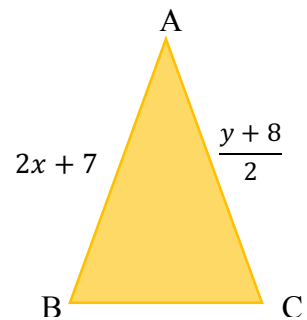
- Form an inequality in  $x$  if Wen Hui can spend at most \$13 on the fruits.
- Solve the inequality in (a).
- Hence, find the maximum number of oranges Wen Hui can buy.
- If Wen Hui buys the greatest possible number of oranges, how much change will she receive?

8. An object moves at an average speed of  $(x + 1)$  m/s for 20 seconds. It then moves at an average speed of  $(2x - 5)$  m/s for the next 30 seconds.

- From an inequality in  $x$  if the average speed of the object during the 50 seconds is less than 4 m/s.
- Solve the inequality in (a).

9. In the figure,  $\triangle ABC$  is an isosceles triangle with  $AB = AC$ . The lengths of  $AB$ ,  $AC$  and  $BC$  are  $(2x + 7)$  cm,  $\frac{y+8}{2}$  cm and 10 cm respectively

- If  $y = 38$ , find
  - the value of  $x$ ,
  - the perimeter of  $\triangle ABC$ .
- Suppose that the perimeter of  $\triangle ABC$  is not more than 60cm.
  - Form an inequality in  $x$  and solve it.
  - Form an inequality in  $y$  and solve it.



10. In a Mathematics examination paper, there were 10 questions in Section A which carry 1 mark each, 5 questions in Section B which carry 3 marks each and 3 questions in Section C which carry 5 marks each. Eric answered  $3x + 2$ ,  $2x - 1$  and  $x$  questions correctly in Section A, B and C respectively.

- Form an inequality in  $x$  if Eric's score for this Mathematics examination paper was more than 10 but at most 40.
- Solve the inequality in (a) and list the possible values of  $x$ .
- Hence, find the possible score of Eric in this examination.

**INTERMEDIATE**

11. Solve each of the following inequalities and represent the solution on a number line.

(a)  $\frac{3x-4}{5} > \frac{2x+7}{6}$

(b)  $\frac{2(x+5)}{3} \leq \frac{4(x-6)}{5}$

(c)  $\frac{5x-9}{2} + 1 \leq \frac{4x+1}{3}$

(d)  $3 - \frac{x+4}{4} < \frac{7x-5}{3}$

(e)  $\frac{3x+1}{5} - \frac{2-3x}{5} \geq \frac{4x-7}{6}$

(f)  $\frac{10+x}{7} < 4 - \frac{8x-9}{2}$

(g)  $\frac{x+4}{2} + \frac{2x-5}{5} > x - \frac{3x-8}{4}$

(h)  $\frac{x-4}{5} - \frac{9+4x}{3} \geq \frac{x}{8} - \frac{5-3x}{3}$

12. Find the smallest and greatest integer values of  $x$  that satisfy each of the following simultaneous inequalities.

(a)  $2 < \frac{x+2}{3} < 5$

(b)  $-5 \leq \frac{x-4}{5} < 8$

(c)  $0 < \frac{3x+10}{6} \leq 2\frac{1}{2}$

(d)  $6 \leq \frac{4x-9}{2} \leq 14$

(e)  $-1 \leq \frac{2(7+5x)}{7} < 10$

(f)  $-10 < \frac{5(2x-1)}{4} < 20$

13. Solve the following simultaneous inequalities.

(a)  $x - 5 < 8 < x + 4$

(b)  $3x - 8 \leq 10 \leq 2x + 7$

(c)  $9 - x \leq 15 < 21 - x$

(d)  $4(11 - x) < 17 \leq 35 - 2x$

(e)  $\frac{2x-5}{5} < 1 \leq \frac{7+3x}{9}$

(f)  $\frac{2x-9}{3} \leq 6 < \frac{5+x}{2}$

14. Solve the following simultaneous inequalities.

(a)  $2x - 1 < \frac{x+8}{2} \leq x + 3$

(b)  $2(x - 5) \leq \frac{8(x+2)}{3} < 3(x - 5)$

(c)  $\frac{2(x-10)}{3} < 4x - 1 < \frac{7x+3}{2}$

(d)  $\frac{9x-2}{4} < 2x + 7 < \frac{8(x+1)}{3}$

(e)  $\frac{2x-7}{3} \leq \frac{2x-5}{4} \leq \frac{5x-9}{5}$

(f)  $\frac{x-11}{2} < \frac{x+4}{3} \leq \frac{3x-4}{5}$

15. Suppose that  $x$  and  $y$  are integers such that  $1 < x \leq 6$  and  $-8 \leq y < 5$ . Find

- (a) the smallest possible value of  $2x + 3y$ ,
- (b) the greatest possible value of  $3x - 4y$ ,
- (c) the greatest possible value of  $x^2 + y^2$ ,
- (d) the smallest possible value of  $\frac{y}{x}$ ,
- (e) the smallest possible value of  $\frac{x-y}{x}$ .

16. 4 different integers 12, 17,  $x$  and 29 are arranged in ascending order.

- (a) The median of the 4 numbers is more than 19 and at most 22.
  - (i) Form an inequality in  $x$ .
  - (ii) Solve the inequality and list the possible values of  $x$ .
- (b) The mean of the 4 numbers is at least 21 and less than 22.
  - (i) Form an inequality in  $x$ .
  - (ii) Solve the inequality and list the possible values of  $x$ .

17. (a) Make  $x$  the subject of the formula  $2\left(4 - \frac{x}{3}\right) = w$ .

- (b) Given that  $w$  is an integer and  $9 \leq x < 30$ , find the smallest and greatest values of  $w$ .

18. The general term of a sequence is  $T_n = an + b$ . The 5<sup>th</sup> term of the sequence is 16 and the sum of the 6<sup>th</sup> term and the 8<sup>th</sup> term is 52.

- (a) Find the values of  $a$  and  $b$ .
- (b) The  $p^{\text{th}}$  term of the sequences is greater than 101 and at most 130.
  - (i) Form a simultaneous inequality in  $p$  and solve it.
  - (ii) Hence, list all the possible values of  $p$ .

19. Hong Yi has  $(5x + 8)$  two-dollar notes,  $(2x - 5)$  five-dollar notes,  $(2x + 7)$  ten-dollar notes and  $x$  one-dollar coins.

- (a) Express the amount of money Hong Yi has in terms of  $x$ .
- (b) If the total amount of money Hong Yi has is more than \$350 but less than \$520, form a simultaneous inequality in  $x$  and solve it.
- (c) Hence, find
  - (i) the possible values of  $x$ ,
  - (ii) the possible amount of money Hong Yi has.

20. During the Chinese New Year, Mr Lim gave a lumpsum of \$2500 to a charity that supported a group of senior citizens. In addition, each of the senior citizens also received \$80 from Mr Lim as a red packet. Suppose that the red packets given out are worth not less than 90% and not more than 140% of the lump sum. Find

- (a) the possible number of senior citizens who are supported by this charity,
- (b) the total possible value of the red packets that were given out,
- (c) the total possible amount that was donated to the charity and its senior citizens.

## Answers:

1. (a)  $>$  (b)  $\leq$  (c)  $>$  (d)  $\leq$  (e)  $>$   
 (f)  $\geq$  (g)  $>$  (h)  $<$
2. (a)  $x > 7$  (b)  $x < 15$  (c)  $x \geq 2$  (d)  $x > -7$   
 (e)  $x \leq -3$  (f)  $x < 4$  (g)  $x \geq 4$  (h)  $x > -4$
3. (a)(i) 12 (ii)  $-2$  (iii)  $-1$  (iv)  $-1$   
 (b)(i) 3 (ii) 6 (iii)  $-9$  (iv) 4
4. (a)  $14 < x < 19$  (b)  $-3 < x < 4$  (c)  $2 < x < 9$   
 (d)  $x \geq 3$  (e)  $x > 24$  (f)  $3 \leq x < 9$   
 (g)  $-1 \leq x < 3$  (h)  $-1 \leq x \leq 7$
5. (a) smallest = 5, greatest = 10  
 (b) smallest = 3, greatest = 7  
 (c) smallest =  $-6$ , greatest = 8  
 (d) smallest = 9, greatest = 13  
 (e) smallest = 5, greatest = 50  
 (f) smallest = 0, greatest = 4  
 (g) smallest = 11, greatest = 120
6. (a) At least 24 marks (b) At least 45 marks
7. (a)  $6 + 0.55x \leq 13$  (b)  $x \leq 12\frac{8}{11}$  (c) 12  
 (d) \$0.40
8. (b)  $x < 4\frac{1}{8}$
9. (a)(i) 8 (ii) 56cm  
 (b)(i)  $x \leq 9$  (ii)  $y \leq 42$
10. (a)  $10 < 14x - 1 \leq 40$  (b)  $x = 1$  or  $x = 2$  (c) 13 or 27
11. (a)  $x > 7\frac{3}{8}$  (b)  $x \geq 61$  (c)  $x \leq 3\frac{2}{7}$  (d)  $x > 1\frac{13}{21}$   
 (e)  $x \geq -1\frac{13}{16}$  (f)  $x < 1\frac{41}{58}$  (g)  $x > 1\frac{7}{13}$  (h)  $x \leq -\frac{256}{271}$
12. (a) smallest = 5, greatest = 12  
 (b) smallest =  $-21$ , greatest = 43  
 (c) smallest =  $-3$ , greatest = 1  
 (d) smallest = 6, greatest = 9  
 (e) smallest =  $-2$ , greatest = 5  
 (f) smallest =  $-3$ , greatest = 8
13. (a)  $4 < x < 13$  (b)  $1\frac{1}{2} \leq x \leq 6$  (c)  $-6 \leq x < 6$   
 (d)  $6\frac{3}{4} < x \leq 9$  (e)  $\frac{2}{3} \leq x < 5$  (f)  $7 < x \leq 13\frac{1}{2}$
14. (a)  $2 \leq x < 3\frac{1}{3}$  (b)  $x > 61$  (c)  $-1.7 < x < 5$   
 (d)  $6\frac{1}{2} < x < 30$  (e)  $1.1 \leq x \leq 6.5$  (f)  $8 \leq x < 41$
15. (a)  $-20$  (b) 50 (c) 100 (d)  $-4$   
 (e)  $-1$
16. (a)(ii)  $x = 22, 23, 24, 25, 26$  or 27  
 (b)(ii)  $x = 26, 27, 28$  or 29
17. (a)  $x = 12 - \frac{3}{2}w$  (b) smallest =  $-11$ , greatest = 2
18. (a)  $a = 5$  and  $b = -9$

- (b)(i)  $22 < p \leq 27.8$  (ii)  $p = 23, 24, 25, 26 \text{ or } 27$
19. (a)  $\$(41x + 61)$  (b)  $7\frac{2}{41} < x < 11\frac{8}{41}$
- (c)(i)  $x = 8, 9, 10 \text{ or } 11$  (ii)  $\$389, \$430, \$471, \$512$
20. (a) 29 to 43 inclusive (b)  $\$2320$  to  $\$3440$  inclusive
- (c)  $\$4820$  to  $\$5940$  inclusive