COMMONWEALTH SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2019

MATHEMATICS
PAPER 2

Name: _____ ( ) Class: _____

SECONDARY FOUR EXPRESS **Friday 30 August 2019**
SECONDARY FOUR NORMAL ACADEMIC (O)
SECONDARY FIVE NORMAL ACADEMIC **14 30 – 17 00**
4048/2 **2h 30min**

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{ , where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{ , where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** Write as a single fraction in its simplest form

(i) $\dfrac{12a^3b}{c^2} \div \dfrac{9ab}{2c^2}$, [1]

(ii) $\dfrac{6}{3-4d} - \dfrac{5}{d+2}$. [2]

(b) Simplify $\dfrac{9-25x^2}{9+6x-15x^2}$. [3]

(c) Solve the equation $3y+2 = \dfrac{10}{y+5}$. [3]

2 **(a)** The exchange rates for Singapore dollars, Swiss Francs and Euro are shown in the table.

Singapore dollars ($) and Swiss Francs (CHF)	$1 = CHF 0.72
Euro (€) and Singapore dollars ($)	€1 = $1.52

Lily is planning a trip to Switzerland and Germany where the currencies used are Swiss Francs and Euro respectively.

She needs to purchase train tickets for her journey from Geneva to Munich and can choose to buy from either SBB or DB website. She finds differences in the price for the same ticket as shown below.

SBB (Switzerland)	CHF 77.00
DB (Germany)	€59.90

Suppose Lily buys from the website offering a better price. She makes her payment using her credit card and the credit card company converts the prices to Singapore dollars. She is charged a currency conversion fee of 1.75%.

Calculate the total amount she has to pay, including the credit card fee. [4]

(b) Mark and Oli have $10 000 each. Mark decides to deposit his money into a bank account which pays 4% compound interest per annum while Oli decides to deposit his money into another bank account which pays 3.2% interest per annum compounded quarterly. Find the percentage difference in the amount of money each of them had after 5 years. [4]

3 A bubble tea store sells green milk tea, oolong milk tea and taro milk tea with the option of no topping, with black pearls or with white pearls. The number of cups of milk tea of each variation being prepared and sold daily are represented by the matrix **N**.

$$\mathbf{N} = \begin{matrix} \text{Green} & \text{Oolong} & \text{Taro} \\ \begin{pmatrix} 25 & 20 & 30 \\ 100 & 85 & 80 \\ 75 & 80 & 70 \end{pmatrix} & & \begin{matrix} \text{No topping} \\ \text{With black pearls} \\ \text{With white pearls} \end{matrix} \end{matrix}$$

(a) A drink with no toppings is sold at \$2.70.
A drink with black pearls is sold at \$3.20.
A drink with white pearls is sold at \$3.50.
Represent these amounts in a row matrix **P**. [1]

(b) Evaluate the matrix **T** = **PN**. [2]

(c) State what the elements of **T** represent. [1]

(d) Write down a matrix **W** such that the product **TW** gives the total earnings of the store for a week. [1]

(e) Suppose the bubble tea store sells each cup for 60% more than its cost price.
On a particular week, the store sold all oolong milk tea drinks at a discount of 10%.
Calculate the total profit made by the store that week. [4]

4 Alex plans to buy x kg of Musang King durians at \$150.

(a) Write down an expression, in terms of x, for the price per kg. [1]

During the durian season, he will be able to buy 2 kg more of Musang King durians with the same amount of money.

(b) Write down an expression for the price per kg during the durian season. [1]

(c) Given that the difference in the price per kg is \$3, write down an equation to represent this information and show that it simplifies to

$$x^2 + 2x - 100 = 0 .$$ [3]

(d) Solve the equation $x^2 + 2x - 100 = 0$, giving your answers correct to 3 decimal places. [3]

(e) Given that a Musang King durian can weigh between 1 kg and 2.7 kg, state the range for the number of durians, y, which Alex would be able to buy during the durian season. [2]

5 **(a)** These are the first four terms in a sequence.

$$-\frac{1}{8} \qquad -\frac{1}{2} \qquad \frac{1}{4} \qquad \frac{1}{10}$$

(i) Find an expression in terms of n for the nth term of the sequence. [2]

(ii) If $\frac{1}{118}$ is a term in the sequence, find the corresponding value of n. [2]

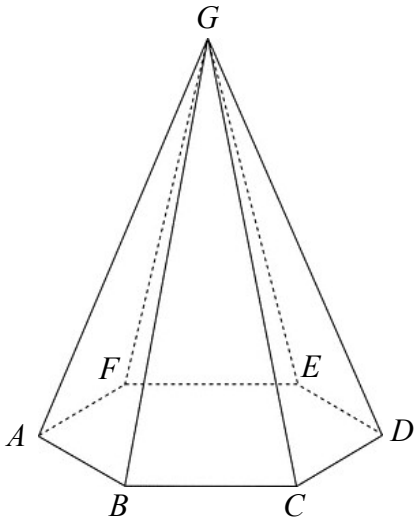
(b) The nth term of another sequence is given by $T_n = n(n+3)$.

(i) Use the formula to find the value of T_7 . [1]

(ii) Given that $16T_7 = T_k$, find k. [3]

(iii) Explain why the difference between two consecutive terms in the sequence cannot be odd. [3]

6



The diagram shows a wooden pyramid *ABCDEFG*.

The base of the pyramid is a regular hexagon of side 10 cm.

G is vertically above the centre of the hexagonal base.

The vertical height of the pyramid is 60 cm.

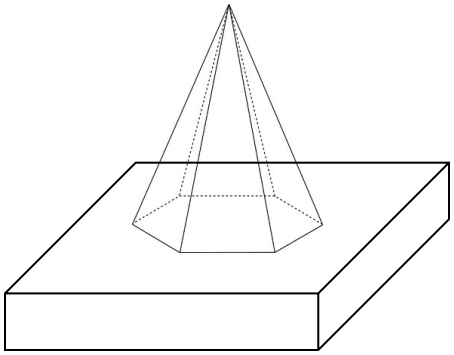
(a) Show that $BG = 60.8$ cm, correct to three significant figures. [2]

(b) Calculate angle *AGB*. [2]

(c) Calculate the total surface area of the pyramid. [3]

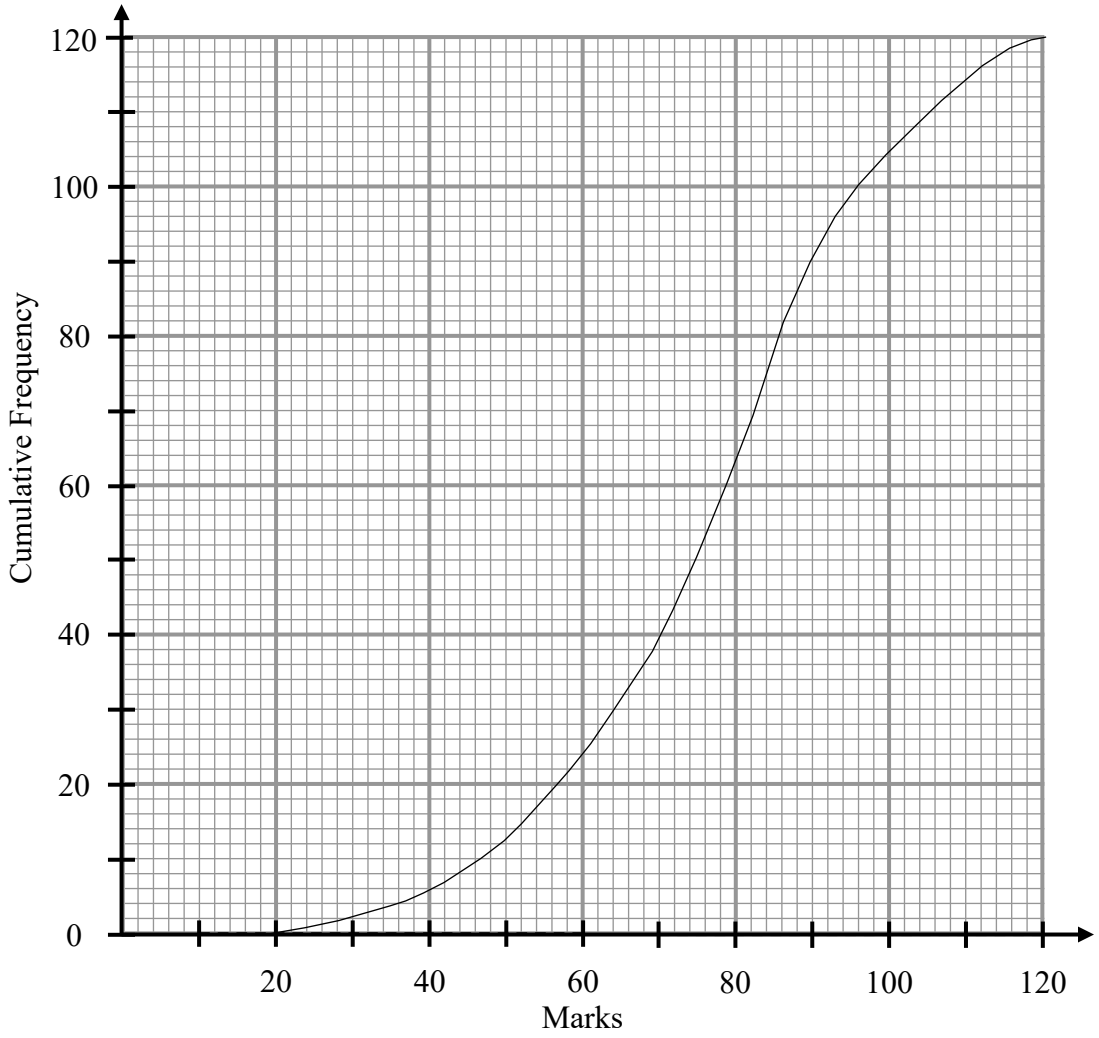
A smaller, similar pyramid of height 16 cm is removed from the top of the original pyramid so that it can be fixed onto a horizontal wooden base with dimensions 55 cm by 45 cm by 6 cm.

The diagram below shows the final product.



(d) Calculate the total surface area of the final product. [4]

7 The cumulative frequency curve below illustrates the marks obtained by 120 students in a Physics examination.



(a) Use the graph to estimate

(i) the 40th percentile, [1]

(ii) the interquartile range. [2]

(b) 2 candidates were chosen at random to participate in a Physics quiz. Calculate the probability that both of them scored above 110 marks in the Physics examination.

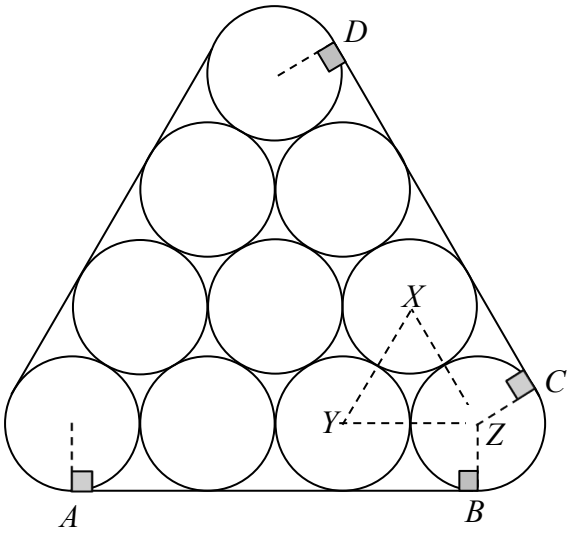
[2]

(c) The same group of students also took a Chemistry examination. The median mark and interquartile range for the Chemistry examination was 79 marks and 33 marks respectively.

Use this information to comment on one difference between the two distributions.

[1]

8



The diagram shows the plan view of 10 toy billiard balls held in place by a wooden holder. Each spherical ball has a radius of 2 cm. The three sides of the holder are tangents to the balls, while the three vertices fit the balls neatly at the corners.
X, Y and Z are the centres of the circles on the diagram.

(a) State the special name for the triangle XYZ. [1]

(b) Find the length of minor arc BC. [2]

(c) Hence find the perimeter of the wooden holder. [2]

On the diagram, AB produced and DC produced meet at a point E.

(d) Show that triangle ZBE and triangle ZCE are congruent.
Give a reason for each statement you make. [3]

(e) Show that $BE = 2\sqrt{3}$ cm. [1]

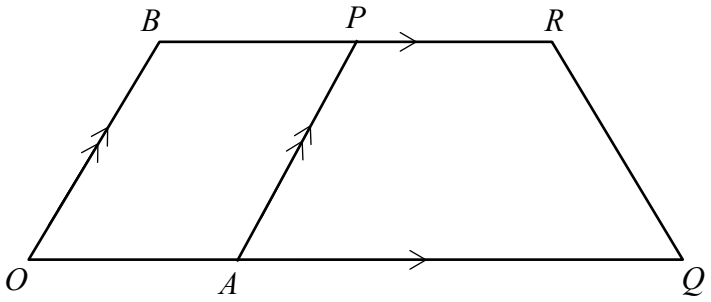
(f) Calculate the total area of the space bounded by the wooden holder that is unoccupied by the billiard balls. [4]

9 **(a)** $\overrightarrow{LM} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ and the coordinates of a point N are $(-3,15)$.

L, M and N are collinear points and L lies on the x axis.

Find the position vector of point L. [4]

(b)



In the diagram, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

$OAPB$ is a parallelogram and $OQRB$ is a trapezium.

P is the midpoint of BR and the ratio of $OA : AQ = 1 : 2$.

(i) Express each of the following in terms of $\mathbf{a}$ and $\mathbf{b}$.

(a) $\overrightarrow{OQ}$, [1]

(b) $\overrightarrow{QP}$, [1]

(c) $\overrightarrow{RQ}$. [1]

(ii) The line OB and QP produced meet at X.

Find $\overrightarrow{QX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]

(iii) Find the numerical value of

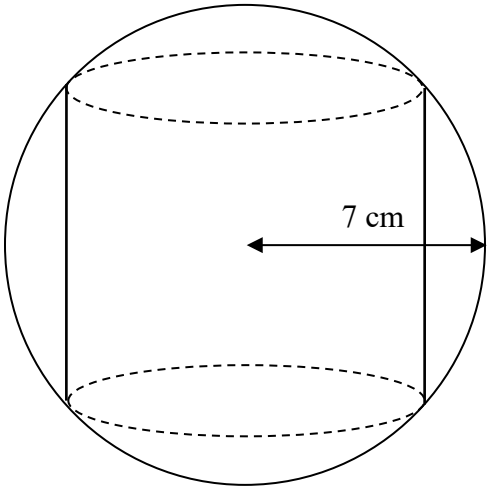
(a) $\dfrac{\text{Area of triangle } BPX}{\text{Area of triangle } OQX}$, [2]

(b) $\dfrac{\text{Area of parallelogram } OAPB}{\text{Area of trapezium } OQRB}$. [1]

10 **Answer the whole of this question on graph paper.**

Mark is working on an art installation. He wants to include a solid cylinder as part of his installation.

He has a solid wooden sphere of radius 7 cm left behind from a previous project and will carve the cylinder out of it such that the rim of the top and bottom of the cylinder are in contact with the surface of the sphere.



By varying the height, h, of the cylinder, Mark can change the volume of the cylinder.

(a) Show that the volume of the solid cylinder is given by $\frac{\pi h}{4}\left(196 - h^2\right)$. [2]

(b) Explain why $0 < h < 14$. [1]

(c) Mark wants the solid cylinder to have a volume of 700 cm^3.
He wants the curved surface area of the cylinder to be as small as possible.
By drawing a suitable graph, work out the curved surface area of the cylinder which he will carve. [8]

END OF PAPER

2019 4E5N Prelim Mathematics Paper 2 Solutions

1ai.	$\frac{12a^3b}{c^2} \div \frac{9ab}{2c^2}$ $= \frac{8a^2}{3}$
1aii.	$\frac{6}{3-4d} - \frac{5}{d+2}$ $= \frac{6(d+2)-5(3-4d)}{(3-4d)(d+2)}$ $= \frac{26d-3}{(3-4d)(d+2)}$
1b.	$\frac{9-25x^2}{9+6x-15x^2}$ $= \frac{(3-5x)(3+5x)}{3(3+5x)(1-x)}$ $= \frac{3-5x}{3(1-x)}$
1c.	$3y+2 = \frac{10}{y+5}$ $3y^2+17y+10 = 10$ $y(3y+17) = 0$ $y = 0$ or $y = -5\frac{2}{3}$
2a.	CHF 1.00 = $\$\left(\frac{1}{0.72}\right)$ SBB Price (in \$) = $\$\left(\frac{77}{0.72}\right)$ $\approx$ \$106.94 DB Price (in \$) = $\$(59.90 \times 1.52)$ $\approx$ \$91.05 Total amount Lily has to pay = $\$91.05 \times \frac{101.75}{100}$ = \$92.64 (2d.p.)
2b.	Amount after 5 years (Mark) = $\$10000\left(1+\frac{4}{100}\right)^5$ = \$12166.53 (2d.p.) Amount after 5 years (Oli) = $\$10000\left(1+\frac{3.2 \div 4}{100}\right)^{5\times 4}$ = \$11727.64 (2d.p.) Percentage difference = $\frac{12166.53-11727.64}{11727.64} \times 100\%$ = 3.74 % (3s.f.)

3a.	$\mathbf{P} = \begin{pmatrix} 2.70 & 3.20 & 3.50 \end{pmatrix}$
3b.	$\mathbf{T} = \begin{pmatrix} 2.70 & 3.20 & 3.50 \end{pmatrix} \begin{pmatrix} 25 & 20 & 30 \\ 100 & 85 & 80 \\ 75 & 80 & 70 \end{pmatrix}$ $= \begin{pmatrix} 650 & 606 & 582 \end{pmatrix}$
3c.	The elements of **T** represent the earnings from the sale of green milk tea, oolong milk tea and taro milk tea respectively.
3d.	$\mathbf{W} = \begin{pmatrix} 7 \\ 7 \\ 7 \end{pmatrix}$
3e.	Cost price $= \frac{100}{160}\begin{pmatrix} 650 & 606 & 582 \end{pmatrix}$ $= \begin{pmatrix} 406.25 & 378.75 & 363.75 \end{pmatrix}$ Selling price after discount for oolong milk tea $= \begin{pmatrix} 650 & 606 \times 0.9 & 582 \end{pmatrix}$ $= \begin{pmatrix} 650 & 545.4 & 582 \end{pmatrix}$ Total profit per drink type $= \begin{pmatrix} 650 & 545.4 & 582 \end{pmatrix} - \begin{pmatrix} 406.25 & 378.75 & 363.75 \end{pmatrix}$ $= \begin{pmatrix} 243.75 & 166.65 & 218.25 \end{pmatrix}$ Total profit made by the store that week = 7($243.75 + $166.65 + $353.59) = $4400.55
4a.	$\$\frac{150}{x}$/kg
4b.	$\$\frac{150}{x+2}$/kg
4c.	$\frac{150}{x} - \frac{150}{x+2} = 3$ $150(x+2) - 150x = 3x(x+2)$ $150x + 300 - 150x = 3x^2 + 6x$ $3x^2 + 6x - 300 = 0$ $x^2 + 2x - 100 = 0$ (shown)
4d.	$x^2 + 2x - 100 = 0$ $x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-100)}}{2(1)}$ $= 9.050$ or -11.050 (3 d.p.)
4e.	Since $x > 0$, $x = 9.050$. $x+2 = 11.050$ $\frac{11.050}{2.7} < y < \frac{11.050}{1}$ $5 \le y \le 11$

5ai.	nth term $= \dfrac{1}{-8+(n-1)(6)}$ $= \dfrac{1}{6n-14}$
5aii.	$\dfrac{1}{118} = \dfrac{1}{6n-14}$ $118 = 6n-14$ $n = 22$
5bi.	$T_7 = 70$
5bii.	$16(70) = T_k$ $1120 = k(k+3)$ $k^2 + 3k - 1120 = 0$ $(k-32)(k+35) = 0$ $k = 32$ or $k = -35$ (rejected)
5biii.	$T_{n+1} - T_n$ $= (n+1)(n+4) - n(n+3)$ $= n^2 + 5n + 4 - n^2 - 3n$ $= 2n + 4$ $= 2(n+2)$ Since the difference between 2 consecutive terms in the sequence is a multiple of 2, the difference must be even and hence cannot be odd.

6a.	By Pythagoras' Theorem, $BG = \sqrt{60^2 + 10^2}$ $\approx$ 60.828 = 60.8 cm (3s.f.)
6b.	Using Cosine rule, $10^2 = \left(\sqrt{3700}\right)^2 + \left(\sqrt{3700}\right)^2 - 2\left(\sqrt{3700}\right)\left(\sqrt{3700}\right)\cos\angle AGB$ $\angle AGB =$ 9.4° (1d.p.) OR $\sin\left(\frac{1}{2}\angle AGB\right) = \frac{\frac{1}{2}AB}{BG}$ $\sin\left(\frac{1}{2}\angle AGB\right) = \frac{5}{\sqrt{3700}}$ $\angle AGB = 2\sin^{-1}\left(\frac{5}{\sqrt{3700}}\right)$ $= 9.4°\ (1\text{d.p.})$
6c.	Total surface area of pyramid $= 6\left[\left(\frac{1}{2}\right)(10)(10)\sin 60° + \left(\frac{1}{2}\right)\left(\sqrt{3700}\right)\left(\sqrt{3700}\right)\sin 9.4300°\right]$ $\approx 150\sqrt{3} + 1818.65$ $= 2080\ \text{cm}^2$ (3s.f.)
6d.	Surface area of cone including hexagonal base $= \left(\frac{16}{60}\right)^2 \times 2078.46$ $\approx 147.80\ \text{cm}^2$ Area of hexagonal base $= \left(\frac{16}{60}\right)^2 \left(150\sqrt{3}\right)$ $= 18.475\ \text{cm}^2$ Surface area of wooden base including hexagon $= 2(55)(45) + (55 + 45 + 55 + 45)(6)$ $= 6150\ \text{cm}^2$ Surface area of final product $= 147.80 + 6150 - 2(18.475)$ $= 6260\ \text{cm}^2$ (3s.f.)

7ai.	74 marks
7aii.	Interquartile range $= 90 - 64$ $= 26$ marks
7b.	P(both students scored above 110 marks) $= \left(\frac{6}{120}\right)\left(\frac{5}{119}\right)$ $= \frac{1}{476}$
7c.	The results for the Physics examination were more consistent as seen by the smaller interquartile range of 26 marks for Physics as compared to 33 marks for Chemistry.
8a.	Equilateral
8b.	Length of minor arc BC $= 2\left(2\pi - \frac{\pi}{2} - \frac{\pi}{2} - \frac{\pi}{3}\right)$ $= \frac{4\pi}{3}$ or 4.19 cm
8c.	Perimeter of the wooden holder $= 3\left(\frac{4\pi}{3}\right) + 3(12)$ ≈ 48.566 $= 48.6 \text{ cm}^2$ (3s.f.)
8d.	ZE is a common side. $ZB = ZC$ (radius) $BE = CE$ (tangents from external point) By SSS Test, triangle ZBE and triangle ZCE are congruent.
8e.	$\tan 60° = \frac{BE}{2}$ $BE = 2\sqrt{3}$ cm (shown)
8f.	Total area bounded the wooden holder unoccupied by billiard balls $= \frac{1}{2}\left(12 + 4\sqrt{3}\right)\left(12 + 4\sqrt{3}\right)\sin 60° - 10\left(\pi \times 2 \times 2\right) - 6\left(\frac{1}{2} \times 2\sqrt{3} \times 2\right) - \pi(2)^2$ ≈ 21.256 $= 21.3 \text{ cm}^2$ (3s.f.) Other methods: $\frac{1}{2}(12)(12)\sin 60° + 3(2)(12) + \pi(2)^2 - 10\pi(2)^2$ or $9\left[\frac{1}{2}(4)(4)\sin 60° - \frac{1}{2}\pi(2)^2\right] + 9\left[(4)(2) - \frac{1}{2}\pi(2)^2\right]$

9a.	$\overrightarrow{ON} = \begin{pmatrix} -3 \\ 15 \end{pmatrix}$ Let $\overrightarrow{OL} = \begin{pmatrix} x_L \\ 0 \end{pmatrix}$ and $L(x_L, 0)$. $\overrightarrow{LM} = k\overrightarrow{LN}$ $\overrightarrow{LM} = k\left(\overrightarrow{ON} - \overrightarrow{OL}\right)$ $\begin{pmatrix} 3 \\ -5 \end{pmatrix} = k\left[\begin{pmatrix} -3 \\ 15 \end{pmatrix} - \begin{pmatrix} x_L \\ 0 \end{pmatrix}\right]$ $\begin{pmatrix} 3 \\ -5 \end{pmatrix} = k\begin{pmatrix} -3 - x_L \\ 15 \end{pmatrix}$ $\Rightarrow \quad -5 = 15k$ $k = -\frac{1}{3}$ $\Rightarrow \quad 3 = -\frac{1}{3}(-3 - x_L)$ $x_L = 6$ $\therefore$ The position vector of L is $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$.
9bia.	$\overrightarrow{OQ} = 3\mathbf{a}$
9bib.	$\overrightarrow{QP} = \mathbf{b} - 2\mathbf{a}$
9bic.	$\overrightarrow{RQ} = \mathbf{a} - \mathbf{b}$
9bii.	Since $\overrightarrow{OQ} = 3\overrightarrow{BP}$, $\overrightarrow{QX} = \frac{3}{2}\overrightarrow{QP}$ $= \frac{3}{2}(\mathbf{b} - 2\mathbf{a})$
9biiia.	$\frac{\text{Area of triangle } BPX}{\text{Area of triangle } OQX} = \left(\frac{1}{3}\right)^2$ $= \frac{1}{9}$
9biiib	$\frac{\text{Area of parallelogram } OAPB}{\text{Area of trapezium } OQRB} = \frac{2}{5}$

10a.

Let radius of the cylinder be r cm.

By Pythagoras' Theorem

$$r^2 = 7^2 - \left(\frac{h}{2}\right)^2$$

$$= 49 - \frac{h^2}{4}$$

Volume of the cylinder

$$= \pi\left(49 - \frac{h^2}{4}\right)h$$

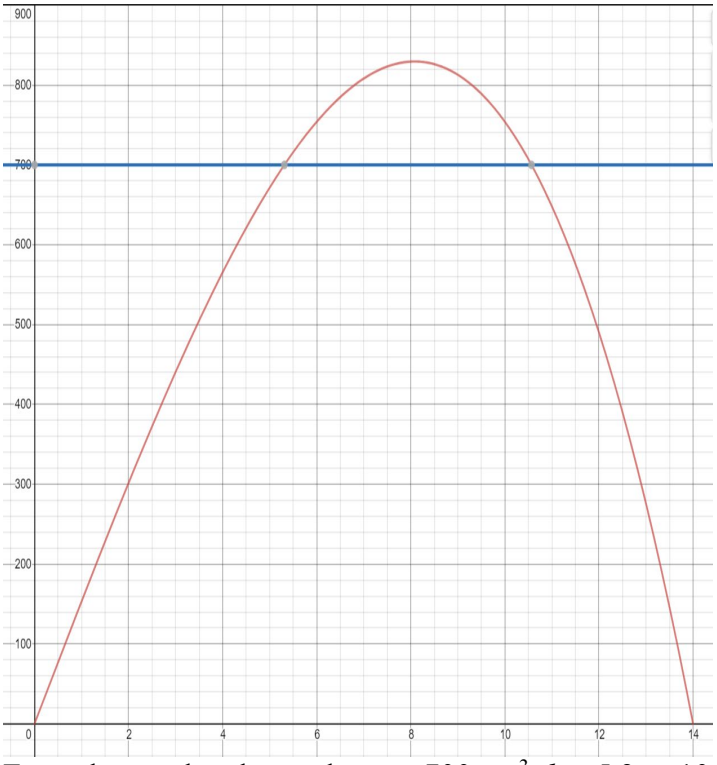
$$= \frac{\pi h}{4}\left(196 - h^2\right)$$

10b.

Radius of the sphere is 7 cm => Diameter of the sphere is 14 cm.
Hence for the cylinder to exist, h must be between 0 cm and 14 cm.

10c.

V	302	565	754	829	754	490
h	2	4	6	8	10	12



From the graph, when volume = 700 cm^3, h = 5.3 or 10.6

Curved surface area of cylinder = $2\pi\left(\sqrt{49 - \frac{h^2}{4}}\right)h$

When h = 5.3, Curved surface area of cylinder = 216 cm^2
When h = 10.6, Curved surface area of cylinder = 305 cm^2

The curved surface area of the cylinder which he will carve is 216 cm^2.