

<b>Name:</b>	<b>Register No.:</b>	<b>Class:</b>
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**CRESCENT GIRLS' SCHOOL  
SECONDARY FOUR  
PRELIM EXAMINATION**

**MATHEMATICS**

**Paper 1**

**4048/01**

**26 August 2019**

**2 hours**

Candidates answer on the Question Paper.

**READ THESE INSTRUCTIONS FIRST**

Write your name and index number on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is **80**.

**For Examiner's Use**

<b>Question</b>	1	2	3	4	5	6	7	8	9	10	11	12
<b>Marks</b>												
<b>Question</b>	13	14	15	16	17	18	19	20	21	22	23	
<b>Marks</b>												

<b>Table of Penalties</b>		<b>Qn. No.</b>	
<b>Presentation</b>	-1		<b>80</b>
<b>Units</b>	-1		
<b>Significant Figures</b>	-1		
			<b>Parent's/ Guardian's Signature</b>

**This question paper consists of 22 printed pages.**

## Mathematical Formulae

### Compound Interest

$$\text{Total amount} = P \left( 1 + \frac{r}{100} \right)^n$$

### Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

### Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

### Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

1 Solve  $3(a - 4) = 1 - (3 - a)$ .

*Answer*  $a = \dots\dots\dots$  [2]

---

2  $\xi = \{\text{quadrilaterals with at least one pair of parallel sides}\}$

$A = \{\text{quadrilaterals with diagonals that bisect each other}\}$

$B = \{\text{quadrilaterals with perpendicular diagonals}\}$

(a) Name a quadrilateral in the set  $A \cap B$ .

*Answer*  $\dots\dots\dots$  [1]

(b) Name a quadrilateral in the set  $(A \cup B)'$ .

*Answer*  $\dots\dots\dots$  [1]

---

3  $n$  is a positive integer.

Show that, for all  $n$ ,  $(7n + 2)^2 - (7n - 2)^2$  is a multiple of 4.

*Answer*

[2]

---

- 4 The cleaning time,  $t$  hours, for a swimming pool is inversely proportional to the square of the number of workers,  $x$ , involved in the cleaning.

If 4 men can complete the cleaning of the swimming pool in 8 hours, how long does it take 8 men to clean the same swimming pool?

*Answer* ..... hours [2]

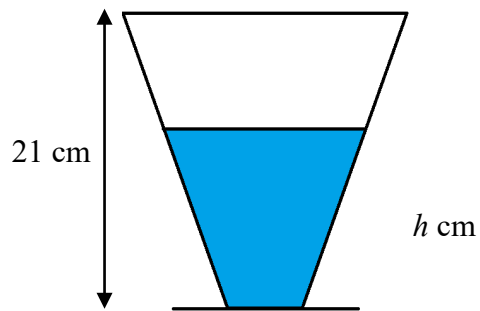
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- 5 The area of a triangle  $PQR$  is  $2360 \text{ m}^2$ .  
 $PQ = 84 \text{ m}$  and  $QR = 65 \text{ m}$ .  
Find the possible sizes of the angle  $PQR$ .

*Answer* ..... or ..... [2]

---

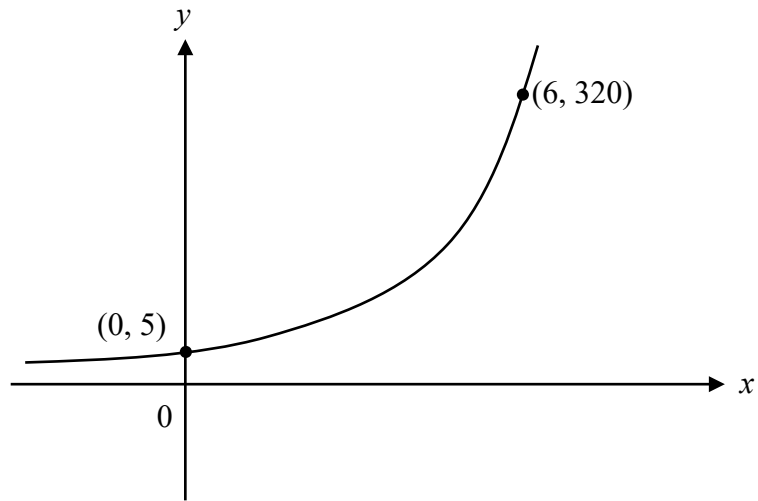
- 6 The diagram shows a styrofoam cup of height 21 cm.



The volume of the liquid in the styrofoam cup is one-third the volume of the cup.  
The radius of the top of the cup is twice that of its base.  
Calculate the depth,  $h$  centimetres, of the liquid in the cup.

*Answer*  $h = \dots\dots\dots$  [2]

- 7 The sketch below shows the graph of  $y = ka^x$ .  
The points  $(0, 5)$  and  $(6, 320)$  lie on the graph.



Find the values of  $k$  and  $a$ .

*Answer*  $k =$  .....  
 $a =$  ..... [2]

- 8 Write as a single fraction in its simplest form  $\frac{5}{(x-4)^2} - \frac{2}{4-x}$ .

*Answer* ..... [3]

- 9 (a) Express 9801 as a product of its prime factors.

*Answer* ..... [1]

- (b) Using your answer to **part (a)**, explain why 9801 is a perfect square.

*Answer*

[1]

- (c)  $c$  and  $d$  are both prime numbers.

Find the smallest values of  $c$  and  $d$  such that  $9801 \times \frac{c}{d}$  is a perfect cube.

*Answer*  $c =$  .....  
 $d =$  ..... [1]

- 10 (a) Express  $23 - 6x + x^2$  in the form  $p + (x + q)^2$ .

*Answer* ..... [2]

- (b) Without sketching the graph, explain why the minimum of  $y = 23 - 6x + x^2$  is 14.

*Answer*

[1]

- 11 Find the values of  $m$  and  $n$  if  $\frac{5^{n+1} - 5^{n-1}}{4 \times 5^{2n}} = m \times 5^{2n}$  and  $m$  is an integer.

*Answer*  $m =$  .....

$n =$  ..... [3]



12 A straight line  $L$  is parallel to the line  $5y = -12x$  and passes through the point  $D(5, 0)$ .

(a) Find the equation of the line  $L$ .

*Answer* ..... [2]

(b) Point  $E$  is the intersection point of line  $L$  and the  $y$ -axis.

Verify that the coordinates of  $E$  is  $(0, 12)$ .

*Answer*

[1]

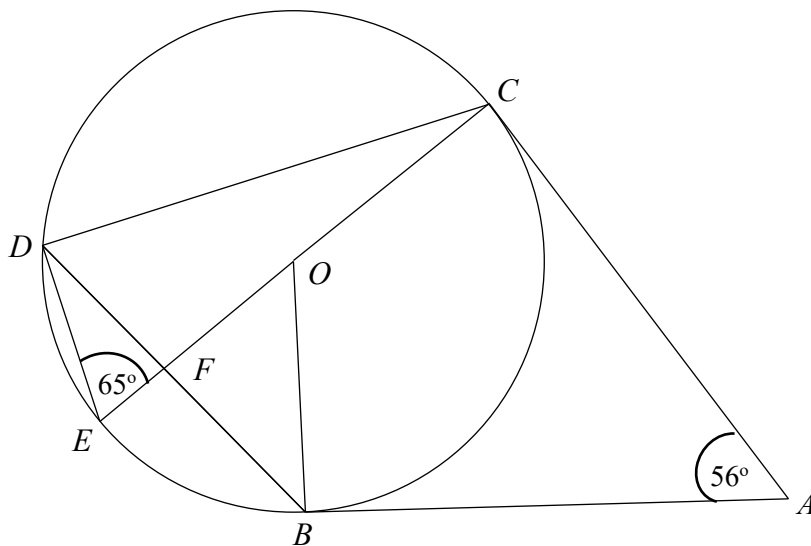
(c) Find the distance between the points  $D$  and  $E$ .

*Answer* ..... [1]

(d) Point  $D$  lies 3 units to the left of point  $F$ .  
Find the value of  $\cos \angle FDE$ .

*Answer* ..... [2]

13



The circle  $BCDE$  has centre  $O$  and  $COFE$  is a straight line.  
 $AB$  and  $AC$  are tangents to the circles.  
 Angle  $CED = 65^\circ$  and angle  $CAB = 56^\circ$ .

(a) Find angle  $BOC$ .

Answer ..... [1]

(b) Find angle  $DFO$ .

Answer ..... [3]

14 A Singapore map has a scale of 1 : 30 000 .

- (a) The length of a segment of the Singapore River on the map is 7 cm.  
Calculate the actual length, in kilometres, of the segment of the Singapore River.

*Answer* ..... km [2]

- (b) The area of a new housing district is 7 km<sup>2</sup>.  
Calculate the area, in square centimetres, of the new housing district on the map.

*Answer* ..... cm<sup>2</sup> [2]

- 15 Alaska earned \$50 000 profit from his business and decided to deposit the whole sum in the bank to earn interest for three years.  
There are two types of plans as shown below.

Advanced Lifestyle Plan: Simple Interest of 4.7% per annum

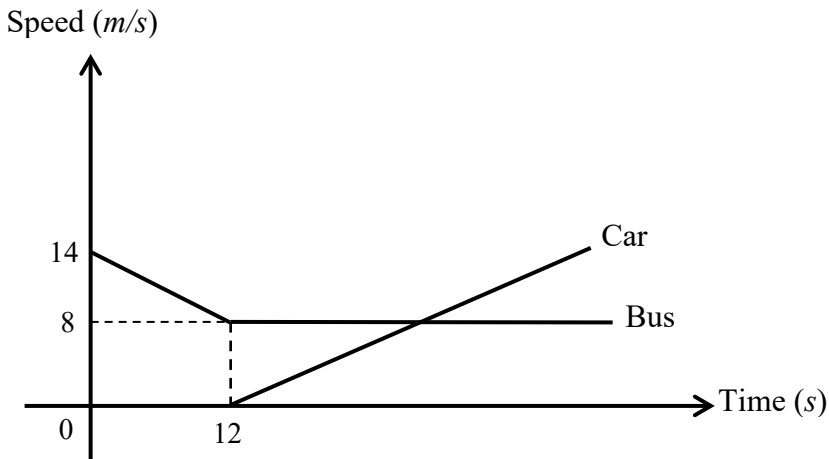
Premium Deposit Plan: Compound Interest of 4.5% per annum compounded quarterly

Determine with mathematical workings to advise Alaska of the choice he should make.

*Answer*

[3]

- 16 The diagram below shows the speed-time graphs of a bus and a car.



The bus decelerated uniformly from 14 m/s to 8 m/s for the first 12 seconds and continued to travel at a constant speed for the rest of the journey.

- (a) Calculate the speed of the bus after it travelled for 5 seconds.

*Answer* ..... m/s [2]

- (b) The car left the same place as the bus after 12 seconds and accelerated uniformly at  $0.5 \text{ m/s}^2$ .

The car meets the bus at time  $t$  seconds.

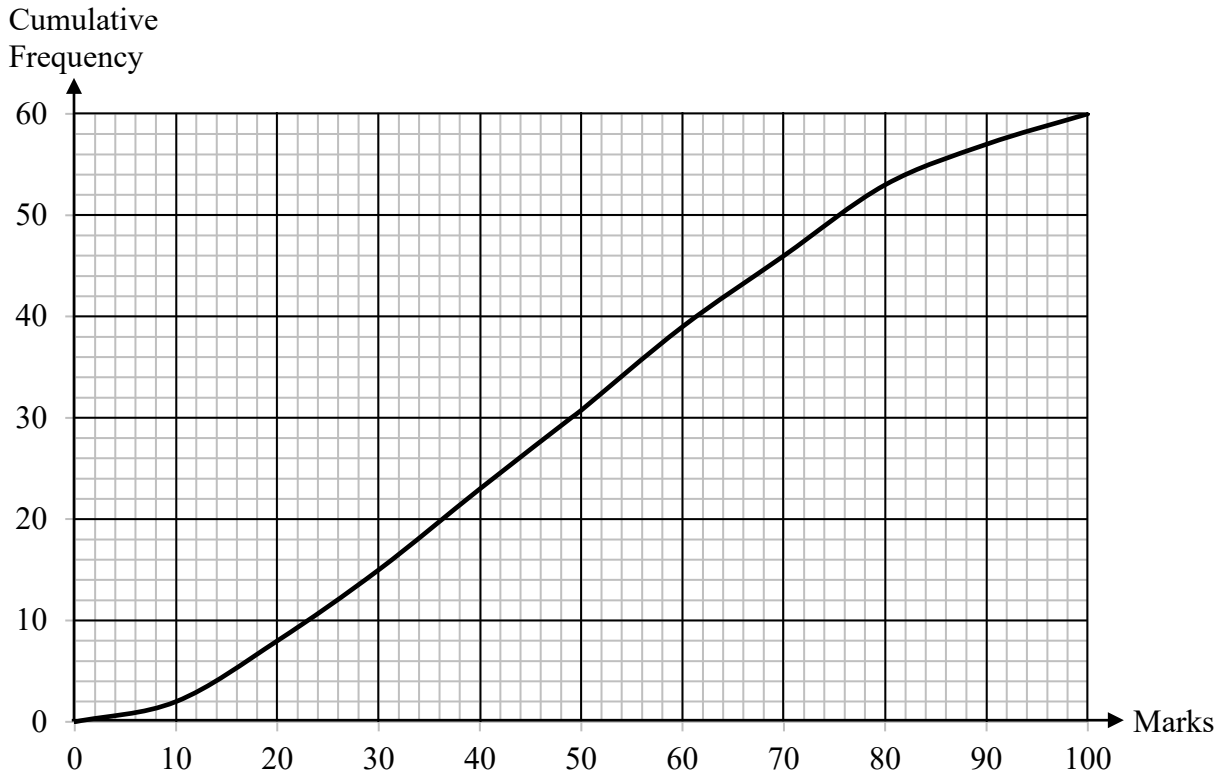
- (i) Write down an equation to represent the information and show that it simplifies to  $t^2 - 56t = 0$ .

*Answer*

- (ii) Calculate the time taken by the car for it to catch up with the bus. [2]

*Answer* ..... seconds [2]

- 17 The cumulative frequency curve shows the marks obtained by 60 students from school *ABC* in a Mathematics Olympiad competition.



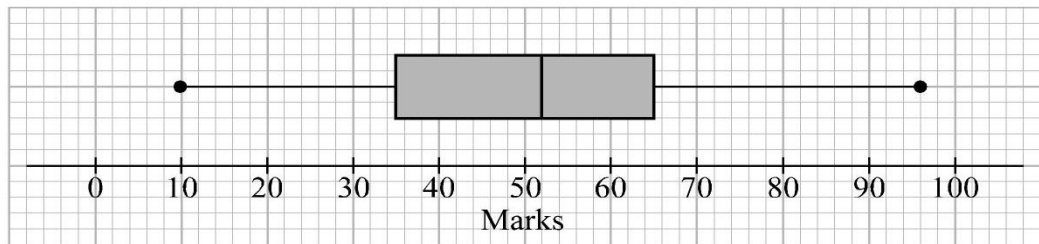
- (a) (i) Find the interquartile range.

*Answer* ..... marks [1]

- (ii) Find the minimum number of marks for the merit award if 30% of the students obtained a merit award.

*Answer* ..... marks [1]

- (b) School *XYZ* also had 60 students who sat for the same Mathematics Olympiad competition as School *ABC*.  
The box and whisker diagram below shows the marks obtained for the students in School *XYZ*.



Write down two statements with reasonings to compare the results of the Mathematics Olympiad competition of the two schools.

*Answer*

[2]

- 18 (a) Solve the inequalities  $\frac{3}{2}x - \frac{28}{3} < 2x - 7 \leq 11 - x$  .

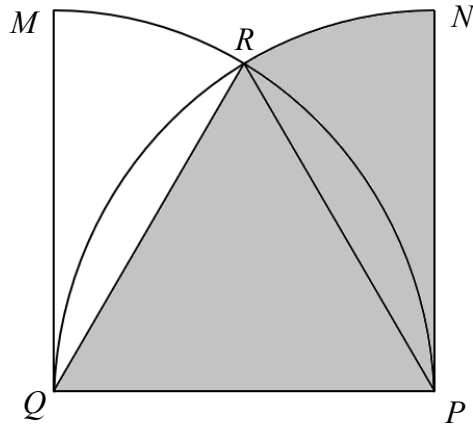
*Answer* ..... [2]

- (b) If  $p$  and  $q$  are two integer values of  $x$  found in (a), find the least value of  $\frac{1}{p^2 - q^2}$  .

*Answer* ..... [1]



19



$PQM$  and  $PQN$  two identical quadrants of a circle.  
The length of  $PQ$  is 8 cm.

(a) Find the perimeter of the shaded region.

Answer ..... cm [2]

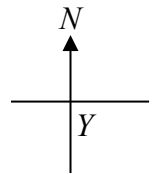
(b) Find the area of the shaded region.

Answer .....  $\text{cm}^2$  [2]

20 In a park  $XYZ$ ,  $XY = 100$  km and  $YZ = 45$  km.

$X$  is due west of  $Y$  and the bearing of  $Z$  from  $Y$  is  $330^\circ$ .

- (a) The point  $Y$  is marked in the space below. [2]  
Construct the park  $XYZ$  using the scale of  $1 \text{ cm} : 10 \text{ km}$ .
- (b) Construct the perpendicular bisector of  $XY$ . [1]
- (c) Construct the bisector of angle  $ZXY$ . [1]
- (d) A shelter is to be built in the park, equidistant from  $X$  and  $Y$ . [1]  
It will be nearer to  $XZ$  than  $XY$ .  
Using a cross, indicate a possible location of the shelter in the diagram.



21  $\overline{AB} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ .

(a) Find  $|\overline{AB}|$ .

*Answer* ..... units [1]

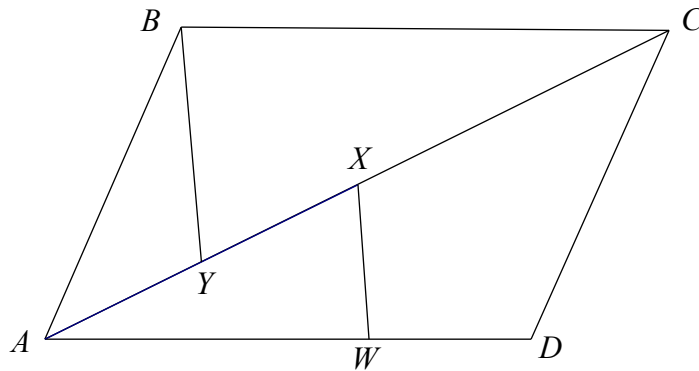
(b)  $C$  is the point  $(9, -1)$ .

$$\overline{DC} = 3\overline{AB}.$$

Find the coordinates of  $D$ .

*Answer* ( ..... , ..... ) [2]

- 22 In the diagram,  $ABCD$  is a parallelogram.  
 $X$  is the midpoint of  $AC$  and  $Y$  is the midpoint of  $AX$ .  
 $W$  is the point on  $AD$  such that  $AW = 2WD$ .  
 $\overline{AB} = \mathbf{p}$  and  $\overline{AD} = 3\mathbf{q}$ .



- (a) Write each of the following in terms of  $\mathbf{p}$  and  $\mathbf{q}$ .  
 Give your answers in their simplest form.

(i) Verify that  $\overline{AC} = \mathbf{p} + 3\mathbf{q}$ .

*Answer*

[1]

(ii)  $\overline{BY}$ .

*Answer* ..... [1]

(iii)  $\overline{XW}$ .

*Answer* ..... [2]

- (b) (i) Explain why  $BY$  is parallel to  $XW$ .

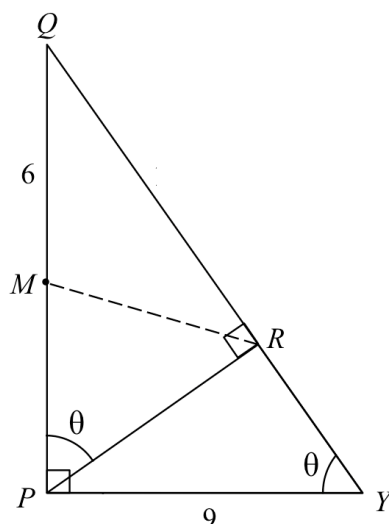
*Answer*

[1]

- (ii) Find the ratio  $BY : XW$ .

*Answer* ..... : ..... [1]

23



The diagram above shows triangle  $QPY$  where angle  $QPY = 90^\circ$  and  $PY = 9$  cm.  $M$  is the midpoint of  $PQ$  and angle  $QPR = \text{angle } PYR = \theta$ .  $QM = 6$  cm and  $R$  is a point on  $QY$  such that angle  $QRP = 90^\circ$ .

- (a) Prove that triangle  $PRY$  and triangle  $QRP$  are similar.

*Answer*

[2]

- (b) Find the area of triangle  $PRY$  : area of triangle  $QRP$ .

*Answer* ..... : ..... [2]

- (c) Find the area of triangle  $QMR$  : area of triangle  $PRY$ .

*Answer* ..... : ..... [2]

**End of Paper**

## ANSWER KEY

Question	Answer
1	$a = 5$
2(a)	Rhombus/ Square
2(b)	Trapezium
3	$(7n+2)^2 - (7n-2)^2 = (7n+2+7n-2)(7n+2-7n+2)$ $= (14n)(4)$ <p>Since <math>n</math> is a positive integer, <math>(7n+2)^2 - (7n-2)^2</math> is a multiple of 4.</p>
4	2 hours
5	$59.8^\circ$ (1dp) or $120.2^\circ$ (1dp)
6	$h = 10.4$
7	$k = 5$ $a = 2$
8	$\frac{2x-3}{(x-4)^2}$
9(a)	$9801 = 3^4 \times 11^2$
9(b)	9801 is a perfect square as $3^4 \times 11^2$ can also be expressed as $(3^2 \times 11)^2$ .
9(c)	$c = 11$ $d = 3$
10(a)	$14 + (x-3)^2$
10(b)	$(x-3)^2 \geq 0$ for all real $x$ $14 + (x-3)^2 \geq 14$ $\therefore$ minimum value of $y = 14$
11	$m = 6$ and $n = -\frac{1}{3}$
12(a)	$y = -\frac{12}{5}x + 12$
12(b)	<p>On <math>y</math>-axis, <math>x = 0</math>, <math>y = 12</math></p> <p>Coordinate of <math>E = (0, 12)</math></p>
12(c)	13 units
12(d)	$-\frac{5}{13}$
13(a)	$124^\circ$
13(b)	$93^\circ$
14(a)	2.1 km
14(b)	$77.8 \text{ cm}^2$
15	Premium Deposit Plan
16(a)	11.5 m/s
16(b)(i)	-
16(b)(ii)	44 s
17(a)(i)	39 marks
17(a)(ii)	64 marks
17(b)	<p>On average, school <math>XYZ</math> did better than school <math>ABC</math> as the median for School <math>XYZ</math> is 52 marks which is higher than the median for School <math>ABC</math> at 49 marks.</p> <p>The marks obtained by students in School <math>XYZ</math> has less variations as the interquartile range for School <math>XYZ</math> is 30 marks as compared to the interquartile range for School <math>ABC</math> of 38 marks.</p>
18(a)	$-4\frac{2}{3} < x \leq 6$

18(b)	-1
19(a)	28.2 cm
19(b)	44.5 cm <sup>2</sup>
20	Construction
21(a)	5.83 units
21(b)	(-6, -10)
22(a)(i)	-
22(a)(ii)	$-\frac{3}{4}(\mathbf{p} - \mathbf{q})$
22(a)(iii)	$-\frac{1}{2}(\mathbf{p} - \mathbf{q})$
22(b)(i)	$\overrightarrow{BY} = \frac{3}{2}\overrightarrow{XW}$ $\therefore BY$ is parallel to $XW$
22(b)(ii)	$BY : XW = 3 : 2$
23(a)	-
23(b)	9 : 16
23(c)	8 : 9



Name: <b>Marking Scheme</b>	Register No.:	Class:
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## Mathematical Formulae

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$$\text{Total amount} = P \left( 1 + \frac{r}{100} \right)^n$$

### *Mensuration*

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$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

### *Trigonometry*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

### *Statistics*

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

1 Solve  $3(a-4) = 1 - (3-a)$ .

$$3(a-4) = 1 - (3-a)$$

$$3a - 12 = 1 - 3 + a \quad \text{M1}$$

$$2a = 10$$

$$a = 5 \quad \text{A1}$$

Answer  $a =$  ..... [2]

---

2  $\xi = \{\text{quadrilaterals with at least one pair of parallel sides}\}$

$$A = \{\text{quadrilaterals with diagonals that bisect each other}\}$$

$$B = \{\text{quadrilaterals with perpendicular diagonals}\}$$

(a) Name a quadrilateral in the set  $A \cap B$ .

Rhombus / Square B1

Answer ..... [1]

(b) Name a quadrilateral in the set  $(A \cup B)'$ .

Trapezium B1

Answer ..... [1]

---

3  $n$  is a positive integer.

Show that, for all  $n$ ,  $(7n+2)^2 - (7n-2)^2$  is a multiple of 4.

Answer

$$(7n+2)^2 - (7n-2)^2 = (7n+2+7n-2)(7n+2-7n+2) \quad \text{M1}$$

$$= (14n)(4)$$

Since  $n$  is a positive integer,  $(7n+2)^2 - (7n-2)^2$  is a multiple of 4. A1

[2]

---

- 4 The cleaning time,  $t$  hours, for a swimming pool is inversely proportional to the square of the number of workers,  $x$ , involved in the cleaning.

If 4 men can complete the cleaning of the swimming pool in 8 hours, how long does it take 8 men to clean the same swimming pool?

$$t = \frac{k}{x^2}, \text{ where } k \text{ is a constant.}$$

$$8 = \frac{k}{4^2}$$

$$k = 128 \quad \text{M1}$$

$$t = \frac{128}{8^2}$$

$$t = 2 \text{ hours} \quad \text{A1}$$

Alternative:

$$t = \frac{k}{x^2}, \text{ where } k \text{ is a constant.}$$

$$k = tx^2$$

$$8(4^2) = t(8^2) \quad \text{M1}$$

$$t = \frac{16}{8}$$

$$t = 2 \text{ hours} \quad \text{A1}$$

Answer ..... hours [2]

- 5 The area of a triangle  $PQR$  is  $2360 \text{ m}^2$ .  
 $PQ = 84 \text{ m}$  and  $QR = 65 \text{ m}$ .  
 Find the possible sizes of the angle  $PQR$ .

$$\frac{1}{2}(PQ)(QR) \sin \angle PQR = 2360$$

$$\sin \angle PQR = \frac{2360}{\frac{1}{2}(84)(65)}$$

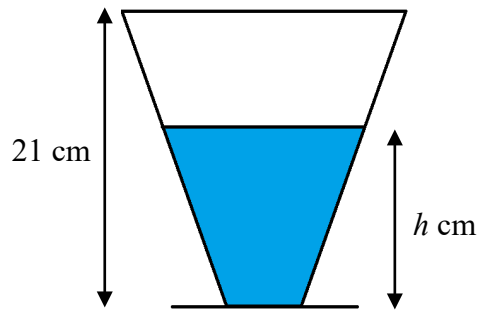
$$\sin \angle PQR = 0.86447 \quad \text{M1}$$

$$\angle PQR = 59.822^\circ \text{ or } \angle PQR = 120.178^\circ$$

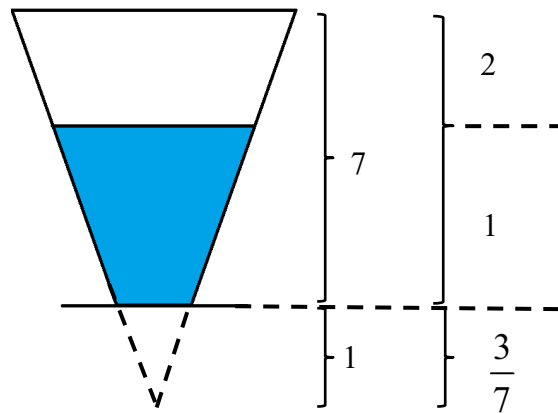
$$= 59.8^\circ \text{ (1dp) or } 120.2^\circ \text{ (1dp)} \quad \text{A1}$$

Answer ..... or ..... [2]

6 The diagram shows a styrofoam cup of height 21 cm.



The volume of the liquid in the styrofoam cup is one-third the volume of the cup.  
 The radius of the top of the cup is twice that of its base.  
 Calculate the depth,  $h$  centimetres, of the liquid in the cup.



$$\left(\frac{h+21}{21+21}\right)^3 = \frac{1\frac{3}{7}}{3\frac{3}{7}} \quad \text{M1}$$

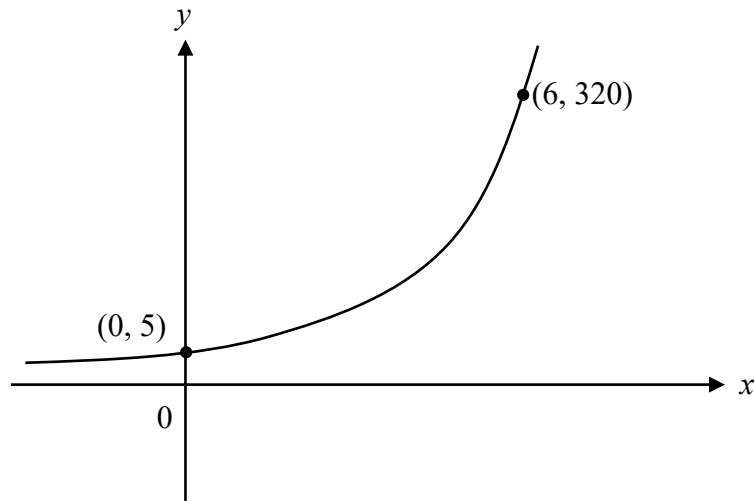
$$\left(\frac{h+21}{42}\right)^3 = \frac{10}{24}$$

$$h = 10.3698$$

$$= 10.4 \text{ cm (3sf)} \quad \text{A1}$$

Answer  $h = \dots\dots\dots$  [2]

- 7 The sketch below shows the graph of  $y = ka^x$ .  
The points  $(0, 5)$  and  $(6, 320)$  lie on the graph.



Find the values of  $k$  and  $a$ .

$$k = 5$$

$$y = 5a^x$$

$$320 = 5a^6$$

$$a = 2 \quad (a > 0)$$

$$\therefore y = 5(2^x)$$

*Answer*  $k =$  .....

$a =$  ..... [2]

- 8 Write as a single fraction in its simplest form  $\frac{5}{(x-4)^2} - \frac{2}{4-x}$ .

$$\frac{5}{(x-4)^2} - \frac{2}{4-x} = \frac{5}{(x-4)^2} + \frac{2}{x-4} \quad \text{M1}$$

$$= \frac{5+2(x-4)}{(x-4)^2} \quad \text{M1}$$

$$= \frac{2x-3}{(x-4)^2} \quad \text{A1}$$

*Answer* ..... [3]

- 9 (a) Express 9801 as a product of its prime factors.

$$9801 = 3^4 \times 11^2 \quad \text{A1}$$

Answer ..... [1]

- (b) Using your answer to **part (a)**, explain why 9801 is a perfect square.

Answer

9801 is a perfect square as  $3^4 \times 11^2$  can also be expressed as  $(3^2 \times 11)^2$ . A1

[1]

- (c)  $c$  and  $d$  are both prime numbers.

Find the smallest values of  $c$  and  $d$  such that  $9801 \times \frac{c}{d}$  is a perfect cube.

$$\begin{aligned} 9801 \times \frac{c}{d} &= 3^4 \times 11^2 \times \frac{c}{d} \\ &= 3^3 \times 11^3 \end{aligned}$$

$$\therefore c = 11 \text{ and } d = 3 \quad \text{A1 for both } c \text{ and } d$$

Answer  $c =$  .....

$d =$  ..... [1]

- 10 (a) Express  $23 - 6x + x^2$  in the form  $p + (x + q)^2$ .

$$\begin{aligned} 23 - 6x + x^2 &= x^2 - 6x + (-3)^2 - (-3)^2 + 23 \quad \text{M1} \\ &= 14 + (x - 3)^2 \quad \text{A1} \end{aligned}$$

Answer ..... [2]

- (b) Without sketching the graph, explain why the minimum of  $y = 23 - 6x + x^2$  is 14.

Answer

$$(x - 3)^2 \geq 0 \text{ for all real } x$$

$$14 + (x - 3)^2 \geq 14$$

$$\therefore \text{minimum value of } y = 14$$

[1]

- 11 Find the values of  $m$  and  $n$  if  $\frac{5^{n+1}-5^{n-1}}{4 \times 5^{2n}} = m \times 5^{2n}$  and  $m$  is an integer.

$$\frac{5^{n+1}-5^{n-1}}{4 \times 5^{2n}} = m \times 5^{2n}$$

$$\frac{5^n \left(5 - \frac{1}{5}\right)}{4 \times 5^n \times 5^n} = m \times 5^{2n}$$

M1

$$\frac{24}{4 \times 5^n} = m \times 5^{2n}$$

$$6 \times 5^{-n-1} = m \times 5^{2n}$$

M1

$$m=6 \quad \text{and} \quad -n-1=2n$$

A1 (for both correct values)

$$n = -\frac{1}{3}$$

Answer  $m =$  .....

$n =$  ..... [3]



12 A straight line  $L$  is parallel to the line  $5y = -12x$  and passes through the point  $D(5, 0)$ .

(a) Find the equation of the line  $L$ .

$$\text{gradient of line} = -\frac{12}{5}$$

At point  $(5, 0)$ ,

$$(y - 0) = -\frac{12}{5}(x - 5) \quad \text{M1}$$

$$y = -\frac{12}{5}x + 12 \quad \text{A1}$$

$$5y = -12x + 60$$

Answer ..... [2]

(b) Point  $E$  is the intersection point of line  $L$  and the  $y$ -axis.

Verify that the coordinates of  $E$  is  $(0, 12)$ .

Answer

On  $y$ -axis,  $x = 0$ ,  $y = 12$  B1

Coordinate of  $E = (0, 12)$

[1]

(c) Find the distance between the points  $D$  and  $E$ .

$$\begin{aligned} \text{distance} &= \sqrt{(5-0)^2 + (0-12)^2} \\ &= 13 \text{ units} \end{aligned} \quad \text{A1}$$

Answer ..... [1]

(d) Point  $D$  lies 3 units to the left of point  $F$ .

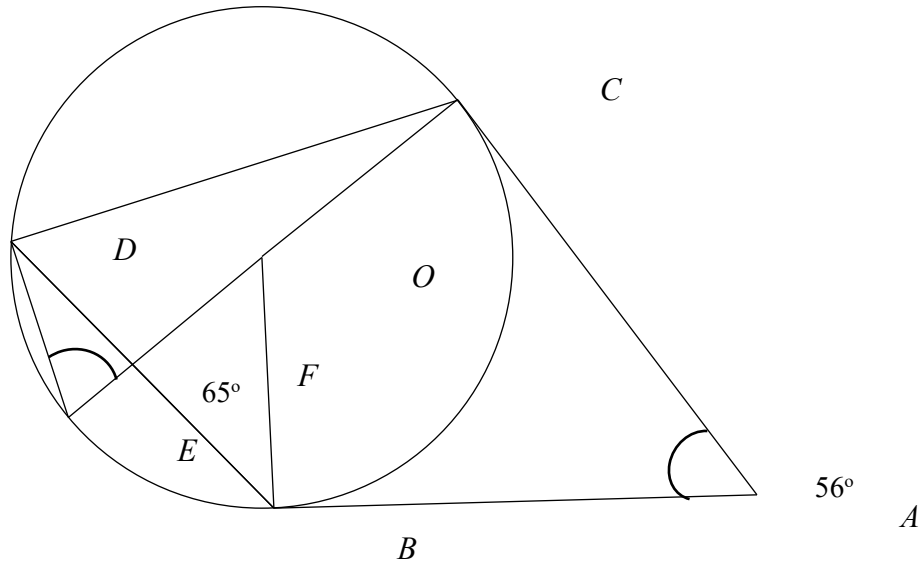
Find the value of  $\cos \angle FDE$ . M1

$$\cos \angle FDE = -\cos \angle EDO$$

$$= -\frac{5}{13} \quad \text{A1}$$

Answer ..... [2]

13



The circle  $BCDE$  has centre  $O$  and  $COFE$  is a straight line.  
 $AB$  and  $AC$  are tangents to the circles.  
 Angle  $CED = 65^\circ$  and angle  $CAB = 56^\circ$ .

(a) Find angle  $BOC$ .

$$\begin{aligned} \text{obtuse angle } BOC &= 360^\circ - 90^\circ - 90^\circ - 56^\circ \text{ (sum of angles in quadrilateral)} \\ &= 124^\circ \quad \text{A1} \end{aligned}$$

Answer ..... [1]

(b) Find angle  $DFO$ .

$$\begin{aligned} \text{angle } BDC &= \frac{124^\circ}{2} \text{ (angle at center} = 2 \times \text{angle at circumference)} \quad \text{M1} \\ &= 62^\circ \end{aligned}$$

$$\begin{aligned} \text{angle } EDF &= 90^\circ - 62^\circ \text{ (angle in semicircle)} \quad \text{M1} \\ &= 28^\circ \end{aligned}$$

$$\begin{aligned} \text{angle } DFO &= 28^\circ + 65^\circ \text{ (external angle of triangle)} \\ &= 93^\circ \quad \text{A1} \end{aligned}$$

Answer ..... [3]

14 A Singapore map has a scale of 1:30 000.

- (a) The length of a segment of the Singapore River on the map is 7cm.  
Calculate the actual length, in kilometres, of the segment of the Singapore River.

1:30000

1 cm : 0.3 km    **M1**

7 cm : 2.1 km    **A1**

Answer ..... km [2]

- (b) The area of a new housing district is 7 km<sup>2</sup>.

Calculate the area, in square centimetres, of the new housing district on the map.

1:30000

1 cm : 0.3 km

1 cm<sup>2</sup> : 0.09 km<sup>2</sup>    **M1**

77.77778 cm<sup>2</sup> : 7 km<sup>2</sup>    **A1**

∴ The area on the map is 77.8 cm<sup>2</sup> (3sf).

Answer ..... cm<sup>2</sup> [2]

- 15 Alaska earned \$50 000 profit from his business and decided to deposit the whole sum in the bank to earn interest for three years.  
There are two types of plans as shown below.

Advanced Lifestyle Plan: Simple Interest of 4.7% per annum

Premium Deposit Plan: Compound Interest of 4.5% per annum compounded quarterly

Determine with mathematical workings to advise Alaska of the choice he should make.

Answer

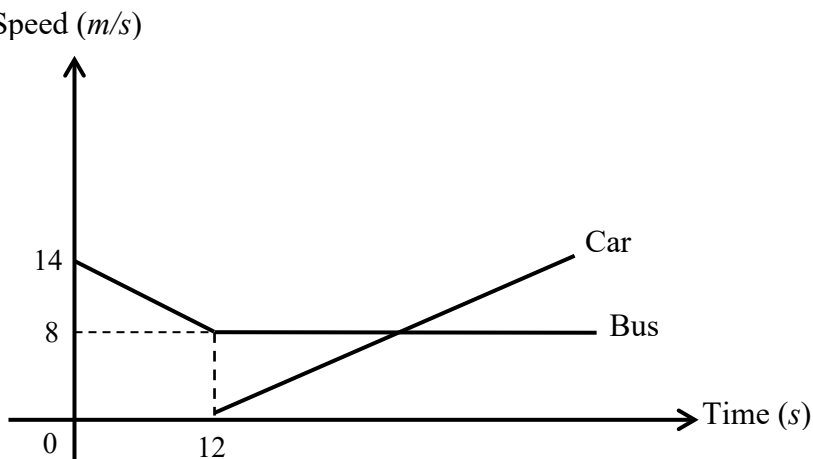
$$\begin{aligned} \text{Amount of money in Advanced Lifestyle Plan} &= 50000 + \frac{50000 \times 4.7 \times 3}{100} \\ &= \$57050 \end{aligned} \quad \text{B1}$$

$$\begin{aligned} \text{Amount of money in Premium Deposit Plan} &= 50000 \left( 1 + \frac{4.5}{100} \right)^{12} \\ &= 57183.72204 \\ &= \$57183.72 \text{ (to 2dp)} \end{aligned} \quad \text{B1}$$

Based on the workings, Alaska should choose Premium Deposit Plan as it will generate a higher return. **B1**

[3]

- 16 The diagram below shows the speed-time graphs of a bus and a car.



The bus decelerated uniformly from 14 m/s to 8 m/s for the first 12 seconds and continued to travel at a constant speed for the rest of the journey.

- (a) Calculate the speed of the bus after it travelled for 5 seconds.

$$\begin{aligned} \text{speed of the bus} &= 14 - 5\left(\frac{6}{12}\right) && \text{M1 for deceleration} \\ &= 11.5 \text{ m/s} && \text{A1} \end{aligned}$$

Answer ..... m/s [2]

- (b) The car left the same place as the bus after 12 seconds and accelerated uniformly at  $0.5 \text{ m/s}^2$ .

The car meets the bus at time  $t$  seconds.

- (i) Write down an equation to represent the information and show that it simplifies to  $t^2 - 56t = 0$ .

Answer

Let the time taken for the bus to meet the car be  $t$ .

$$\frac{1}{2}(6)(12) + 8(t) = \frac{1}{2}(t-12)\left(\frac{1}{2}t - 6\right) \quad \text{M1 for distance of the car}$$

$$36 + 8t = \left(\frac{1}{2}t - 6\right)^2$$

$$36 + 8t = \frac{1}{4}t^2 - 6t + 36$$

$$\frac{1}{4}t^2 - 14t = 0$$

$$t^2 - 56t = 0 \text{ (shown)} \quad \text{A1}$$

[2]

(ii) Calculate the time taken by the car for it to catch up with the bus.

$$t^2 - 56t = 0$$

$$t\left(\frac{1}{4}t - 14\right) = 0$$

$$t = 0 \text{ (reject } \because t > 12) \text{ or } t = 56 \text{ M1 for both solutions (including rejection)}$$

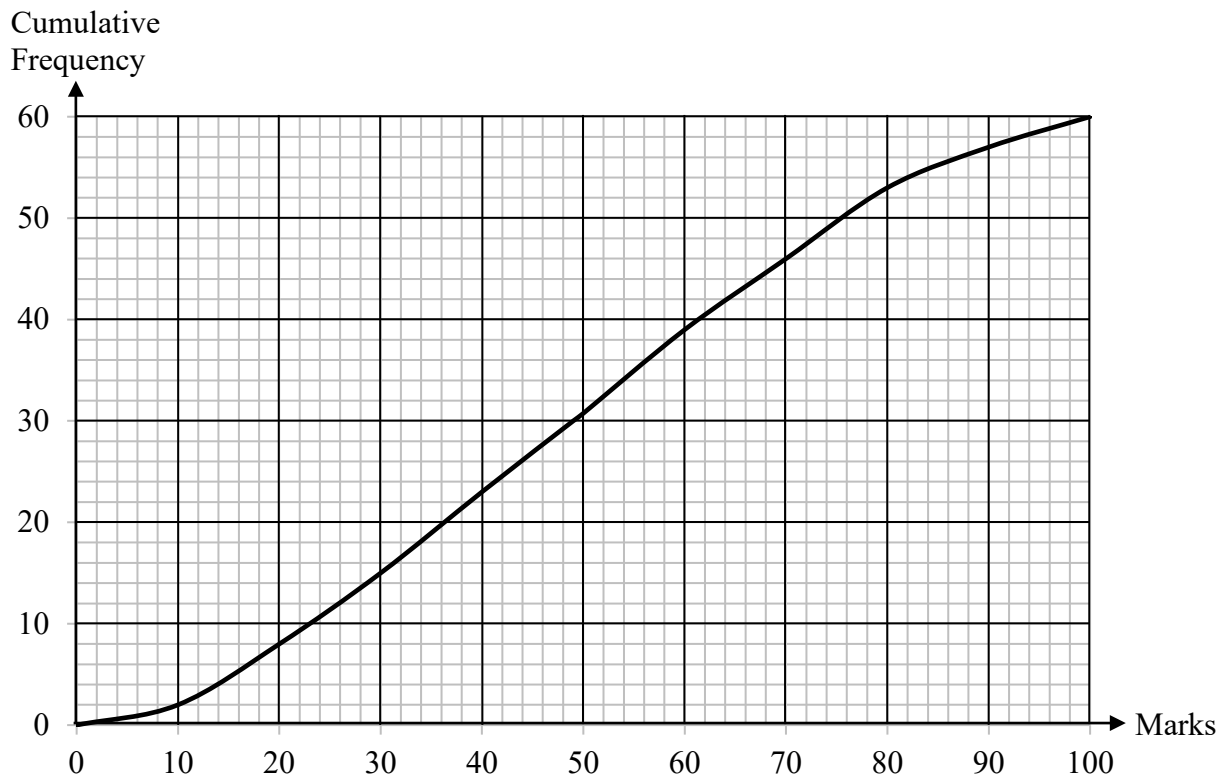
$$\text{time taken by the car} = 56 - 12$$

$$= 44\text{s}$$

**A1**

Answer ..... seconds [2]

- 17 The cumulative frequency curve shows the marks obtained by 60 students from school *ABC* in a Mathematics Olympiad competition.



- (a) (i) Find the interquartile range.  
 $IQR = 69 - 30$   
 $= 39$  marks ( $\pm 1$ )

Answer ..... marks [1]

- (ii) Find the minimum number of marks for the merit award if 30% of the students obtained a merit award.

No. of students who obtained a merit award

$$= 60 \times \frac{30}{100}$$

$$= 18$$

No. of students who did not obtain a merit award

$$= 60 - 18$$

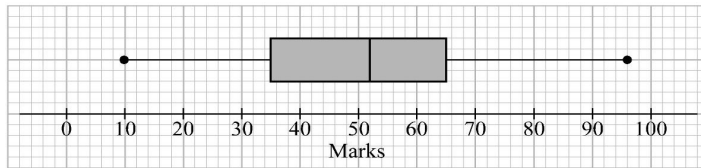
$$= 42$$

No. of marks for merit award

$$= 64$$
 marks

Answer ..... marks [1]

- (b) School *XYZ* also had 60 students who sat for the same Mathematics Olympiad competition as School *ABC*.  
The box and whisker diagram below shows the marks obtained for the students in School *XYZ*.



Write down two statements with reasonings to compare the results of the Mathematics Olympiad competition of the two schools.

*Answer*

On average, school *XYZ* did better than school *ABC* as the median for School *XYZ* [2] is 52 marks which is higher than the median for School *ABC* at 49 marks.

The marks obtained by students in School *XYZ* has less variations as the interquartile range for School *XYZ* is 30 marks as compared to the interquartile range for School *ABC* of 38 marks.

- 18 (a) Solve the inequalities  $\frac{3}{2}x - \frac{28}{3} < 2x - 7 \leq 11 - x$ .

$$\frac{3}{2}x - \frac{28}{3} < 2x - 7 \leq 11 - x$$

$$\frac{3}{2}x - \frac{28}{3} < 2x - 7 \quad \text{and} \quad 2x - 7 \leq 11 - x \quad \text{M1}$$

$$\frac{3}{2}x - \frac{28}{3} < 2x - 7 \quad \quad \quad 3x \leq 18$$

$$9x - 56 < 12x - 42 \quad \quad \quad x \leq 6$$

$$-3x < 56$$

$$x > -4\frac{2}{3}$$

$$\therefore -4\frac{2}{3} < x \leq 6 \quad \text{A1}$$

Answer ..... [2]

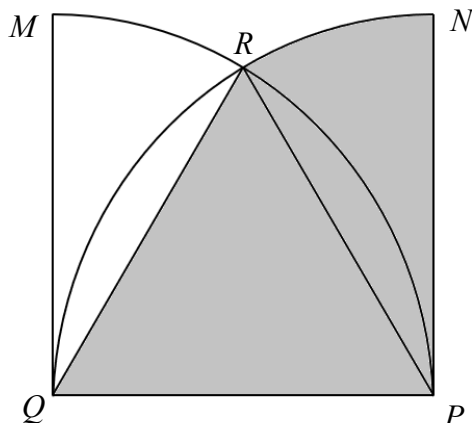
- (b) If  $p$  and  $q$  are two integer values of  $x$  found in (a), find the least value of  $\frac{1}{p^2 - q^2}$ .

$$\frac{1}{(0)^2 - (1)^2} = -1$$

Answer ..... [1]



19



$PQM$  and  $PQN$  two identical quadrants of a circle.  
The length of  $PQ$  is 8 cm.

- (a) Find the perimeter of the shaded region.

$$PQ = QR = PR = 8 \text{ cm (radii of identical quadrants)}$$

$$\begin{aligned} \text{perimeter} &= 8 + 8 + 8 + \frac{30}{360} \times 2\pi(8) \quad \text{M1} \\ &= 28.2 \text{ cm (3sf)} \quad \text{A1} \end{aligned}$$

Alternative:

$$PQ = QR = PR = 8 \text{ cm (radii of identical quadrants)}$$

$$\begin{aligned} \text{perimeter} &= 8 + 8 + 8 + 8 \left( \frac{\pi}{6} \right) \quad \text{M1} \\ &= 28.2 \text{ cm (3sf)} \quad \text{A1} \end{aligned}$$

Answer ..... cm [2]

- (b) Find the area of the shaded region.

$$\begin{aligned} \text{area} &= \frac{1}{2} \times 8 \times (\sqrt{8^2 - 4^2}) + \frac{30}{360} \times \pi(8)^2 \quad \text{M1} \\ &= 44.5 \text{ cm}^2 \text{ (3sf)} \quad \text{A1} \end{aligned}$$

Alternative:

$$\begin{aligned} \text{area} &= \frac{1}{2} \times 8 \times (\sqrt{8^2 - 4^2}) + \left( \frac{1}{2} \right) (8)^2 \left( \frac{\pi}{6} \right) \quad \text{M1} \\ &= 44.5 \text{ cm}^2 \text{ (3sf)} \quad \text{A1} \end{aligned}$$

Answer ..... cm<sup>2</sup> [2]

20 In a park  $XYZ$ ,  $XY = 100$  km and  $YZ = 45$  km.

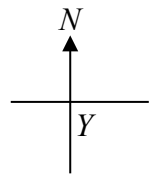
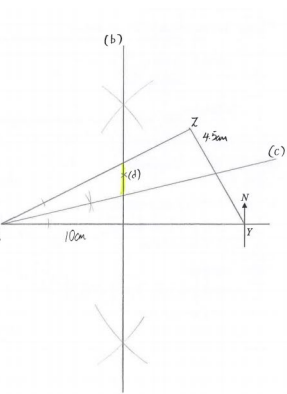
$X$  is due west of  $Y$  and the bearing of  $Z$  from  $Y$  is  $330^\circ$ .

(a) The point  $Y$  is marked in the space below. Construct the park  $XYZ$  using the scale of  $1 \text{ cm} : 10 \text{ km}$ . [2]

(b) Construct the perpendicular bisector of  $XY$ . [1]

(c) Construct the bisector of angle  $ZXY$ . [1]

(d) A shelter is to be built in the park, equidistant from  $X$  and  $Y$ . It will be nearer to  $XZ$  than  $XY$ . Using a cross, indicate a possible location of the shelter in the diagram. [1]



$$21 \quad \overline{AB} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}.$$

(a) Find  $|\overline{AB}|$ .

$$\begin{aligned} |\overline{AB}| &= \sqrt{5^2 + 3^2} \\ &= 5.83 \text{ units (3sf)} \quad \text{A1} \end{aligned}$$

Answer ..... units [1]

(b)  $C$  is the point  $(9, -1)$ .

$$\overline{DC} = 3\overline{AB}.$$

Find the coordinates of  $D$ .

$$\overline{DC} = 3\overline{AB}$$

$$\overline{OC} - \overline{OD} = 3 \begin{pmatrix} 5 \\ 3 \end{pmatrix} \quad \text{M1}$$

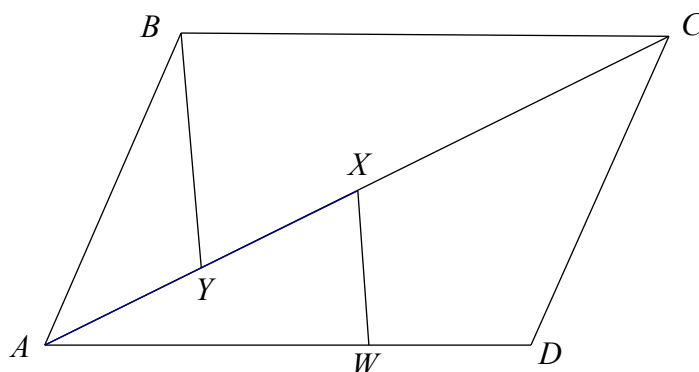
$$\overline{OD} = \begin{pmatrix} 9 \\ -1 \end{pmatrix} - \begin{pmatrix} 15 \\ 9 \end{pmatrix}$$

$$= \begin{pmatrix} -6 \\ -10 \end{pmatrix}$$

$$\therefore D \text{ is } (-6, -10) \quad \text{A1}$$

Answer ( ..... , ..... ) [2]

- 22 In the diagram,  $ABCD$  is a parallelogram.  
 $X$  is the midpoint of  $AC$  and  $Y$  is the midpoint of  $AX$ .  
 $W$  is the point on  $AD$  such that  $AW = 2WD$ .  
 $\overrightarrow{AB} = \mathbf{p}$  and  $\overrightarrow{AD} = 3\mathbf{q}$ .



- (a) Write each of the following in terms of  $\mathbf{p}$  and  $\mathbf{q}$ .

Give your answers in their simplest form.

- (i) Verify that  $\overrightarrow{AC} = \mathbf{p} + 3\mathbf{q}$ .

*Answer*

Since  $ABCD$  is a parallelogram,  $\overrightarrow{BC} = \overrightarrow{AD} = 3\mathbf{q}$ .

$$\begin{aligned}\overrightarrow{AC} &= \overrightarrow{AB} + \overrightarrow{BC} \\ &= \mathbf{p} + 3\mathbf{q} \quad \text{B1}\end{aligned}$$

[1]

- (ii)  $\overrightarrow{BY}$ .

$$\begin{aligned}\overrightarrow{BY} &= \overrightarrow{BA} + \overrightarrow{AY} \\ &= -\mathbf{p} + \frac{1}{4}(\mathbf{p} + 3\mathbf{q}) \\ &= -\frac{3}{4}\mathbf{p} + \frac{3}{4}\mathbf{q} \\ &= -\frac{3}{4}(\mathbf{p} - \mathbf{q}) \quad \text{B1}\end{aligned}$$

*Answer* ..... [1]

- (iii)  $\overrightarrow{XW}$ .

$$\begin{aligned}\overrightarrow{XW} &= \overrightarrow{XA} + \overrightarrow{AW} \\ &= -2\overrightarrow{AY} + \overrightarrow{AW} \\ &= -\frac{1}{2}(\mathbf{p} + 3\mathbf{q}) + 2\mathbf{q} \quad \text{M1} \\ &= -\frac{1}{2}(\mathbf{p} - \mathbf{q}) \quad \text{A1}\end{aligned}$$

*Answer* ..... [2]

- (b) (i) Explain why  $BY$  is parallel to  $XW$ .

*Answer*

$$\overline{BY} = \frac{3}{2} \overline{XW}$$

$\therefore BY$  is parallel to  $XW$

[1]

- (ii) Find the ratio  $BY : XW$ .

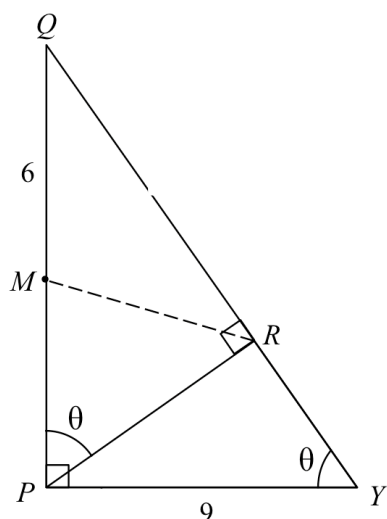
$$\overline{BY} = \frac{3}{2} \overline{XW}$$

$$BY : XW = 3 : 2$$

*Answer* ..... : ..... [1]

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23



The diagram above shows triangle  $QPY$  where angle  $QPY = 90^\circ$  and  $PY = 9$  cm.  $M$  is the midpoint of  $PQ$  and angle  $QPR = \text{angle } PYR = \theta$ .  $QM = 6$  cm and  $R$  is a point on  $QY$  such that angle  $QRP = 90^\circ$ .

- (a) Prove that triangle  $PRY$  and triangle  $QRP$  are similar.

*Answer*

In  $\triangle PRY$  and  $\triangle QRP$ ,

$\angle PRY = \angle QRP = 90^\circ$  (adjacent angles on a straight line)

$\angle QPR = \angle PYR = \theta$  (given)

$\triangle PRY$  is similar to  $\triangle QRP$ .

} M1  
A1

[2]

- (b) Find the area of triangle  $PRY$  : area of triangle  $QRP$ .

Area of triangle  $PRY$  : Area of triangle  $QRP$

$= PY^2 : QP^2$

$= 9^2 : 12^2$      M1

$= 9 : 16$      A1

*Answer* ..... : ..... [2]

- (c) Find the area of triangle  $QMR$  : area of triangle  $PRY$ .

Area of  $\triangle QMR$  : Area of  $\triangle PRY$

$= \frac{1}{2} \times \text{Area of } \triangle QPR : \text{Area of } \triangle PRY$

$= \frac{1}{2} \times 16 : 9$      M1

$= 8 : 9$      A1

*Answer* ..... : ..... [2]

**End of Paper**