Name : __________________________ (   ) Class : Sec 3 / ____ Date : ______

Chapter 6.2 Angles in a Circle
Chapter 6.3 Angles in the Opposite Segment
Chapter 6.4 Tangents of Circles

1. In the diagram, *BOH* is the diameter of the circle with centre *O*. *ABC* and *EDC* are tangents to the circle at *B* and *D* respectively. It is given that $\angle FDG = 22°$, $\angle BFD = 62°$ and $\angle GFD = 77°$.

Stating your reasons clearly, find

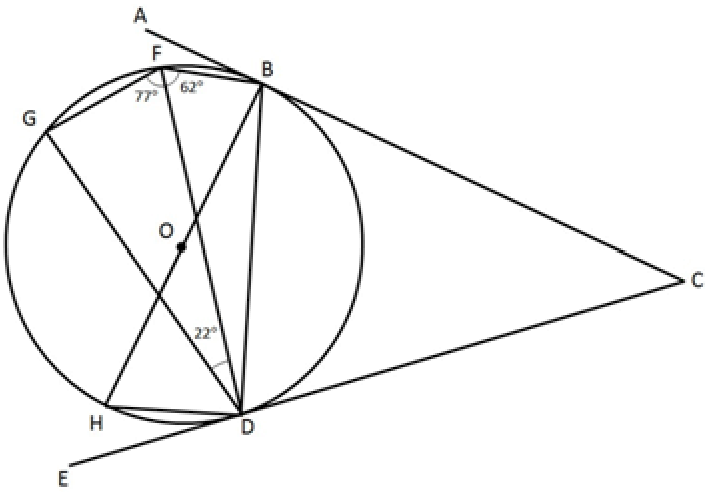
(a) $\angle FOG$

(b) $\angle OGF$

(c) $\angle FDB$

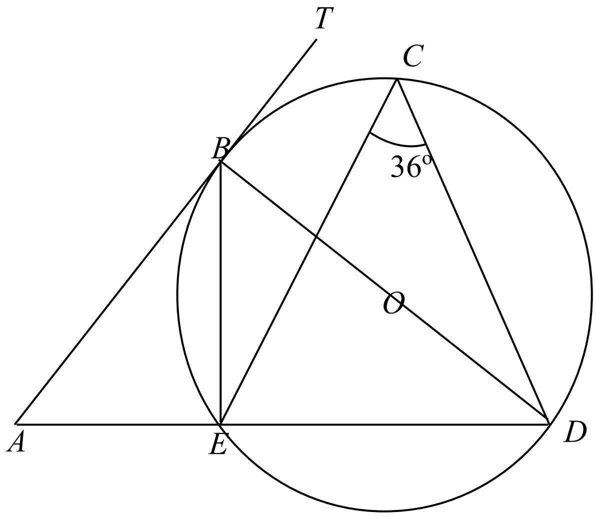
(d) $\angle HBD$

(e) $\angle BCD$



Ans: (a) 44º (b) 68º (c) 19º (d) 28º (e) 56º

2.



In the diagram above, the points *B*, *C*, *D*, *E* lie on a circle with centre *O*. *BD* is the diameter of the circle, *AT* is tangent to the circle at *B*, *AED* is a straight line and $\angle DCE = 36°$. Find

(a) $\angle DBE$,

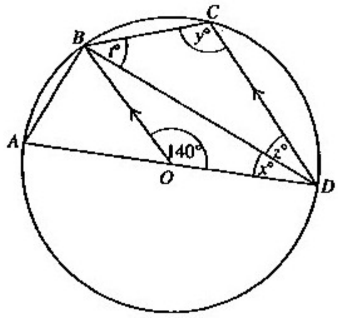
(b) $\angle EOD$,

(c) $\angle BDE$,

(d) $\angle BAD$.

Ans: (a) 36° (b) 72° (c) 54° (d) 36°

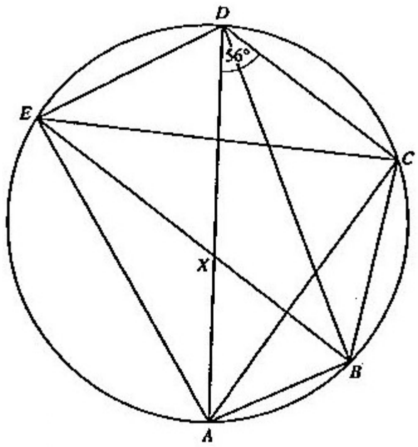
3. In the diagram, the points, A, B, C and D lie on a circle, centre O. AOD is a diameter, OB is parallel to DC and $\angle BOD = 140°$. Find



(a) x, [1]
(b) y, [1]
(c) z, [1]
(d) t. [1]
(N2006/I/19)

Ans: (a) $x = 20$ (b) $y = 110$ (a) $z = 20$ (a) $t = 50$

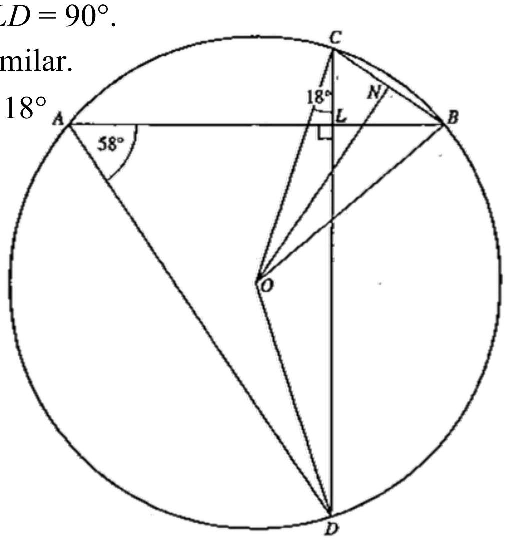
4. The points, A, B, C, D and E lie on a circle. AD is a diameter of the circle. DB bisects $\angle ADC$. Angle $ADC = 56°$.



(a) Giving your reasons, write down
   (i) angle DCA, [1]
   (ii) angle DAC, [1]
   (iii) angle CBA, [1]
   (iv) angle AEB. [1]
(b) It is given that EB is parallel to DC and that EB cuts AD at X.
   [You must not assume that X is the centre of the circle.]
   Prove that triangle BDX is isosceles. [2]
(c) Find angle EBA. [1]
(d) Hence or otherwise prove that X is the centre of the circle. [1]
(N2005/I/3)

Ans: (a)(i) 90° (ii) 34° (iii) 124° (iv) 28° (c) 62°

5. The diagram shows a circle, $ACBD$, with centre O.
The chords AB and CD intersect at L. Angle $ALD = 90°$.



(a) Show that triangles LAD and LCB are similar.
(b) N is the midpoint of BC. Angle $OCD = 18°$ and angle $DAB = 58°$. Find
   (i) angle CNO, [1]
   (ii) angle CON, [1]
   (iii) angle CBA, [1]
   (iv) angle ADO. [1]
(N2008/II/6)

Ans: (b)(i) 90° (ii) 14° (iii) 32° (iv) 14°

Solutions

1.(a) $\angle FOG = 2 \times \angle FDG$ ($\angle$ at centre is twice $\angle$ at circumference)
$= 2 \times 22°$
$= 44°$

1.(b) $\angle OFG = \dfrac{180° - 44°}{2}$ (base $\angle$s of isos ΔOFG)
$= 68°$

1.(c) $\angle FDB = 180° - 77° - 62° - 22°$ ($\angle$s in opp segment)
$= 19°$

1.(d) $\angle BHD = \angle BFD$ ($\angle$s in same segment)
$= 62°$
$\angle BDH = 90°$ ($\angle$ in a semicircle)
$\angle HBD = 180° - 90° - 62°$ ($\angle$ sum of ΔHBD)
$= 28°$

1.(e) $\angle OBC = 90°$ (tangent $\perp$ radius)
$\angle DBC = 90° - 28°$
$= 62°$
$BC = DC$ (tangents from ext pt)
$\angle BDC = \angle DBC = 62°$ (base $\angle$ s of isos ΔBCD)
$\angle BCD = 180° - 62° - 62°$ ($\angle$ sum of ΔBCD)
$= 56°$

2.(a) $\angle DBE = \angle DCE = 36°$ ($\angle$s in same segment)

2.(b) $\angle EOD = 2 \times \angle DCE$ ($\angle$ at centre is twice $\angle$ at circumference)
$= 2 \times 36°$
$= 72°$

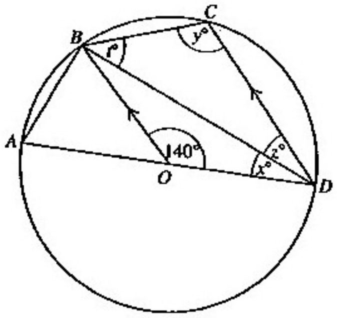
2.(c) $\angle BED = 90°$ ($\angle$ in a semicircle)
$\angle BED = 180° - 90° - 36°$ ($\angle$ sum of ΔBED)
$= 54°$

2.(d) $\angle ABD = 90°$ (tangent $\perp$ radius)
$\angle BAD = 180° - 90° - 54°$ ($\angle$ sum of ΔABD)
$= 36°$

3.(a) $x^\circ = \dfrac{180^\circ - 140^\circ}{2}$ (base $\angle$s of isos ΔOBD)

$= 20^\circ$

$\therefore x = 20$



3.(b) Reflex $\angle BOD = 360^\circ - 140^\circ$ ($\angle$s at a pt)

$= 220^\circ$

$2 \times \angle BCD =$ Reflex $\angle BOD$ ($\angle$ at centre is twice $\angle$ at circumference)

$2y^\circ = 220^\circ$

$y^\circ = 110^\circ$

$\therefore y = 110$

3.(c) $\angle OBD = x^\circ = 20^\circ$ (base $\angle$s of isos ΔOBD)

$z^\circ = \angle OBD = 20^\circ$ (alt $\angle$s, $OB // DC$)

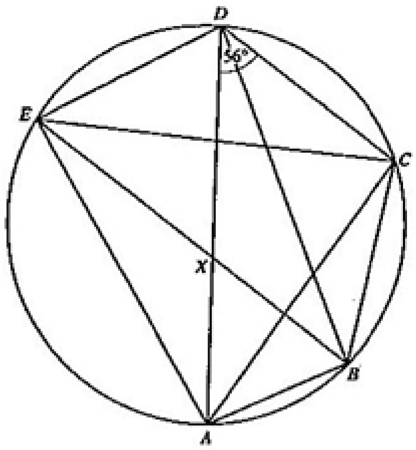
$\therefore z = 20$

3.(d) $t^\circ = 360^\circ - 20^\circ - 140^\circ - 20^\circ - 20^\circ - 110^\circ$ ($\angle$ sum of quad $OBCD$)

$= 50^\circ$

$\therefore t = 50$

4.(a)(i) $\angle DCA = 90^\circ$ ($\angle$ in a semicircle)



4.(a)(ii) $\angle DAC = 180^\circ - 90^\circ - 56^\circ$ ($\angle$ sum of Δ)

$= 34^\circ$

4.(a)(iii) $\angle CBA = 180^\circ - 56^\circ$ ($\angle$s in opp segment)

$= 124^\circ$

4.(a)(iv) $\angle ADB = \angle BDC$ (DC bisects $\angle ADC$)

$= 56^\circ \div 2$

$= 28^\circ$

$\angle AEB = \angle ADB$ ($\angle$s in same segment)

$= 28^\circ$

4.(b) $\angle XBD = \angle BDC = 28^\circ$ (alt $\angle$s, $EB // DC$)

$\angle XDB = \angle ADB = 28^\circ$

Since both base angles $\angle XBD$ and $\angle XDB$ are equal (28°),
therefore, ΔBDX is an isosceles triangle.

4.(c) $\angle DBC = \angle DAC = 34°$ ($\angle$s in same segment)

$\angle EBA = 124° - 28° - 34°$

$= 62°$

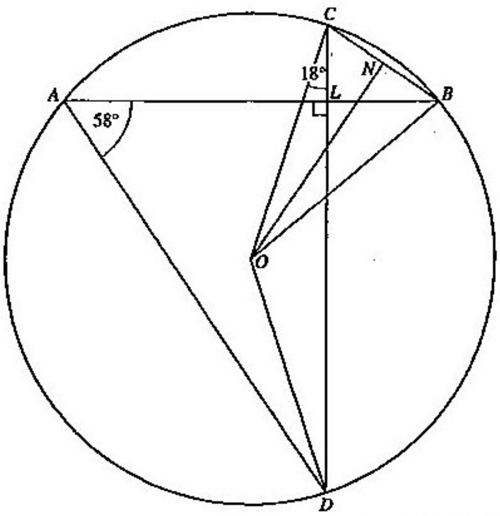
4.(d) $\angle EAB = 180° - 28° - 62°$ ($\angle$ sum of ΔEBA)

$= 90°$

Hence, EB is a diameter $\because \angle$ in a semicircle since $\angle EAB = 90°$

Since X is the intersection point of two diameters, AD and EB, therefore, it is the centre of the circle.

5.(a)



$\angle ALD = \angle CLB = 90°$ (vert opp $\angle$s)

$\angle DAL = \angle BCL = 58°$ ($\angle$s in same segment)

$\therefore \Delta LAD$ is similar to ΔLCB (AA)

5.(b)(i) $\angle CNO = 90°$ ($\perp$ bisector of chord)

5.(b)(ii) $\angle CON = 180° - 90° - 18° - 58°$ ($\angle$ sum of ΔCON)

$= 14°$

5.(b)(iii) $\angle ADC = 180° - 90° - 58°$ ($\angle$ sum of ΔLAD)

$= 32°$

$\angle CBA = \angle ADC = 32°$ ($\angle$s in same segment)

5.(b)(iv) $\angle ODC = 18°$ (base $\angle$s of isos ΔOCD)

$= 32°$

$\angle ADO = 180° - 90° - 58° - 18°$ ($\angle$s sum of ΔLAD)

$= 14°$