



**COMMONWEALTH SECONDARY SCHOOL  
PRELIMINARY EXAMINATION 2019**

**MATHEMATICS  
PAPER 1**

Name: \_\_\_\_\_ (       ) Class: \_\_\_\_\_

**SECONDARY FOUR EXPRESS  
SECONDARY FOUR NORMAL ACADEMIC (O)  
SECONDARY FIVE NORMAL ACADEMIC  
4048/01**

**Tuesday 27 Aug 2019  
10 00 – 12 00  
2h**

**READ THESE INSTRUCTIONS FIRST**

Write your name, index number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

P		-1
A		-1

For Examiner's Use	
Total	80

Parent's Signature: \_\_\_\_\_

This paper consists of **14** printed pages including the cover page.

[Turn over

*Compound interest*

$$\text{Total amount} = P \left( 1 + \frac{r}{100} \right)^n$$

*Mensuration*

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

*Trigonometry*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

*Statistics*

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

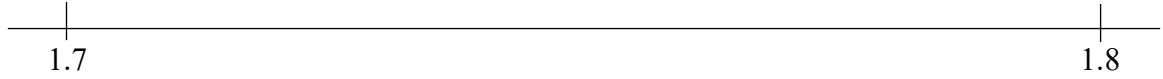
$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

1 Represent the following numbers on a number line.

$$\sqrt{3}, 1.7\dot{3}, 1.733, \frac{7}{4}$$

*Answer*



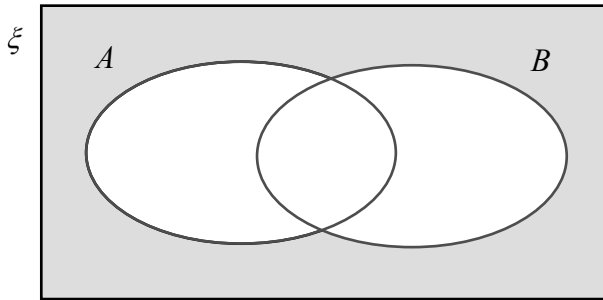
[1]

2 Simplify  $\frac{\sqrt{9y^4}}{(8x^3)^{\frac{2}{3}}}$ .

*Answer* ..... [2]

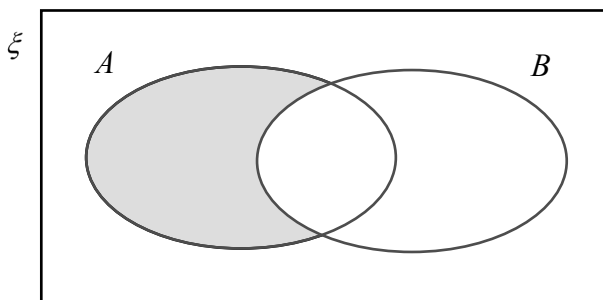
3 Write down the sets represented by the following shaded regions.

(a)



*Answer* ..... [1]

(b)



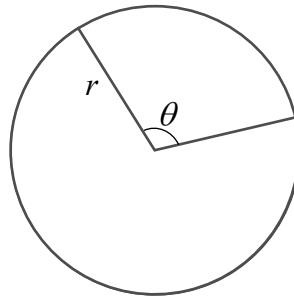
*Answer* ..... [1]

- 4 Show that  $1 - (4n + 3)^2$  is a multiple of 8 for all integer values of  $n$ .

*Answer*

[2]

5



The diagram shows a circle with radius  $r$  cm.  
 The circle is divided into two sectors.  
 The angle of the minor sector is  $\theta$  radians.

The area of the major sector is 150% larger than the area of the minor sector.

Find the value of  $\theta$ , leaving your answers in terms of  $\pi$ .

*Answer* ..... radians [2]

6 One solution of the equation  $kx^2 + (k - 7)x = k - 2$  is  $x = 1$ .  
Find

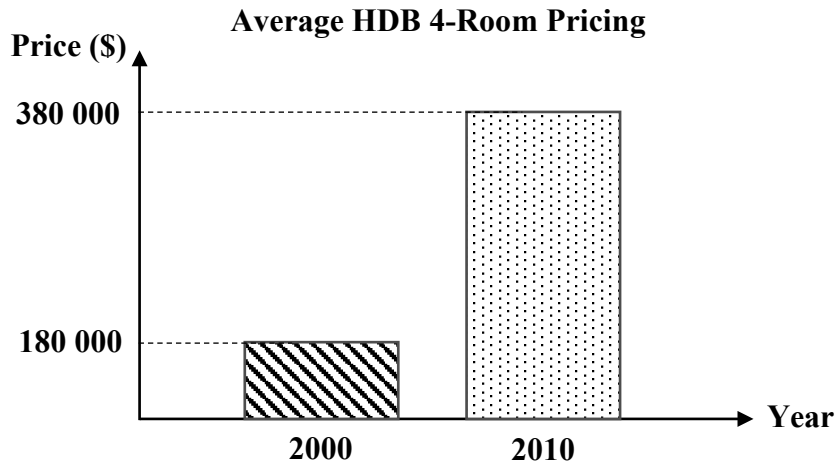
(a) the value of  $k$ ,

Answer  $k = \dots\dots\dots$  [1]

(b) the second possible value of  $x$ .

Answer  $x = \dots\dots\dots$  [1]

7



The graph shows the change in housing prices over the past 10 years.

(a) Explain why this graph may be misleading.

.....  
..... [1]

(b) Suggest how this graph can be better presented to prevent such misinterpretation.

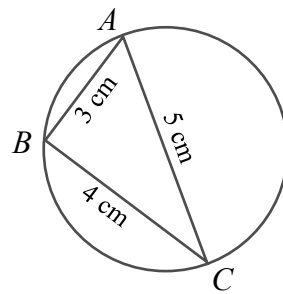
.....  
..... [1]

- 8 The volume of a water tank is  $500 \text{ m}^3$ .  
A model of this tank is built to a scale of 1 : 50

Find the volume, in  $\text{cm}^3$ , of the model.  
Give your answer in standard form.

Answer .....  $\text{cm}^3$  [3]

9



In the diagram,  $A$ ,  $B$  and  $C$  are points on the circle.  
Find the area of the circle.

Answer .....  $\text{cm}^2$  [3]

- 10** A bus leaves Singapore at 05 45 local time.  
There is no time difference between Singapore and Genting, in Malaysia.  
There are two possible routes from Singapore to Genting.

The first route from Singapore to Genting is 400 km.  
On this route, the bus will arrive in Genting at 12 05 local time.

The second route from Singapore to Genting is 467 km.  
On this route, the bus will arrive 30 minutes earlier.

Calculate the difference between the average speeds of the bus on each route.

*Answer* ..... km/h [4]

- 11** Factorise completely

(a)  $4a^5b - 9ab^3$ ,

*Answer* ..... [2]

(b)  $3p^2q^2 - 2pq^2 + 6p - 4$ .

*Answer* ..... [2]

- 12 (a)** The time taken by 2 workers to paint 10 classrooms is 50 hours.

Find the time it would have taken 5 workers to paint the 20 identical classrooms.

*Answer* ..... hours [2]

- (b)** Hooke's law states that, within certain limits, the displacement,  $d$ , of an elastic object is directly proportional to the applied stress,  $s$ , to it.

Calculate the percentage change in an object's displacement when the stress applied is tripled.

*Answer* ..... % [2]

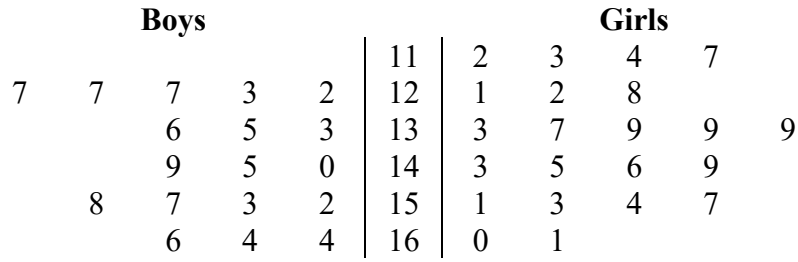
- 13** The mean weight of a box of fruits is 1 kg.  
There are 2 more apples than pears in the box.  
An apple and a pear respectively weighs 75 g and 95 g on average.

Calculate the total number of fruits in the box.

*Answer* ..... [4]



- 14 The heights, in centimetres, of 18 boys and 22 girls were recorded. The results are shown in the stem-and-leaf diagram.



Key (Boys)  
2 | 12 means 122 cm

Key (girls)  
11 | 2 means 112 cm

- (a) Write down the median height of the children.

*Answer* ..... cm [1]

- (b) Calculate the interquartile range for the heights of the children.

*Answer* ..... cm [1]

- (c) Make two comments comparing the heights of the boys and the girls.

1. ....

.....

2. ....

..... [2]

- 15 The points  $(-1,9)$  and  $(2,12)$  lie on the curve given by the equation  $y = 3x^2 + px + q$ . Use an algebraic method to find the values of  $p$  and of  $q$ .

*Answer*  $p =$  .....  
 $q =$  ..... [4]

16 A car is priced at \$97 750.

The hire-purchase price of the car is given a \$2000 discount.

The hire-purchase price is a deposit of 20% plus 60 equal monthly payments with a simple interest rate of 1.8% per annum.

Calculate the total cost of the car paid by hire-purchase.

Answer \$ ..... [4]

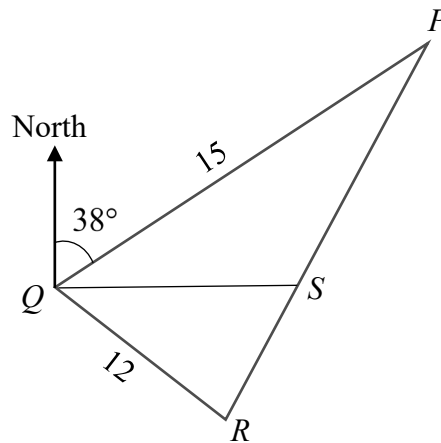
17 The diagram shows the positions of three schools.

$PQ = 15$  km,  $QR = 12$  km and the bearing of  $P$  from  $Q$  is  $038^\circ$ .

$S$  is to the East of  $Q$ .

The area of triangle  $SQR$  is half the area of triangle  $PQR$ .

Find the angle  $SQR$ .



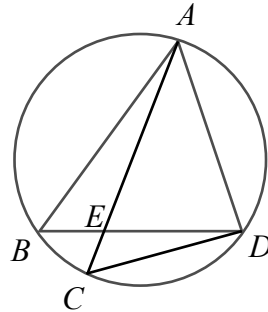
Answer ..... $^\circ$  [4]

- 18 Rearrange the formula  $y = \frac{5-x^2}{x^2+a}$  to make  $x$  the subject.

Answer  $x = \dots\dots\dots$  [3]

- 19 In the diagram,  $A, B, C$  and  $D$  are four points on a circle.

The line  $AC$  and  $BD$  intersect at point  $E$ .



- (a) Show whether triangle  $ABE$  and triangle  $DCE$  are similar.

Answer

[2]

- (b) The area of triangle  $ACD$  is half the product of  $AD$  and  $CD$ .

State, with explanation, if  $AC$  is the diameter of the circle.

Answer

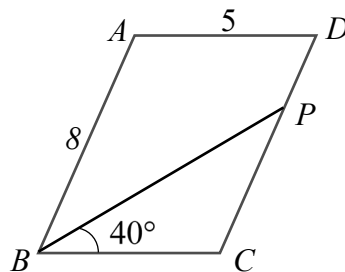
[2]

- 20 The vertices of three regular polygons meet at a point.  
The first polygon is a hexagon.  
The second polygon has twice as many sides as the first polygon.

Name the third polygon.

Answer ..... [4]

21



$ABCD$  is a parallelogram.  
 $BCP$  is an isosceles triangle.  
 $AB = 8$  cm,  $AD = 5$  cm, and angle  $CBP = 40^\circ$ .

- (a) Show that  $BP$  is an angle bisector of angle  $ABC$ .

Answer

[2]

- (b) Calculate the value of  $\frac{\text{Area of } BCP}{\text{Area of } ABPD}$ .

Answer ..... [2]

22 (a) Express 504 as the product of its prime factors.

*Answer* ..... [2]

(b) The number  $504m$  is a perfect cube.  
Find the smallest positive integer of  $m$ .

*Answer*  $m =$  ..... [1]

(c) The highest common factor of  $x$  and 504 is 42.  
Given  $x < 504$ , find the largest value of  $x$ .

*Answer*  $x =$  ..... [3]

23 (a)  $\overline{AB} = \begin{pmatrix} 4k \\ 3k \end{pmatrix}$

$$|\overline{AB}| = 15$$

Find the two possible values of  $k$ .

Answer  $k = \dots\dots\dots$  or  $\dots\dots\dots$  [3]

(b) A line joins the two points  $P(-1,7)$  and  $Q(3,15)$ .

(i) Find  $\overline{PQ}$ .

Answer  $\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$  [1]

(ii) Find the acute angle that the line  $PQ$  makes with the  $x$ -axis.

Answer  $\dots\dots\dots^\circ$  [2]

(iii) Use vectors to show whether or not the point  $R(1,4)$  lies on line  $PQ$ .

Answer

[2]

**End of Paper**

**4E5N MATHEMATICS PRELIMINARY EXAMINATIONS 2019 PAPER 1 SOLUTIONS**

1	
2	$\frac{\sqrt{9y^4}}{(8x^3)^{-\frac{2}{3}}} = 3y^2(8x^3)^{\frac{2}{3}}$ $= 3y^2(4x^2)$ $= 12x^2y^2$
3 (a)	$A' \cap B'$
3 (b)	$A \cap B'$
4	$1 - (4n + 3)^2 = 1 - (16n^2 + 24n + 9)$ $= 8(-2n^2 - 3n - 1)$ <p>Since <math>1 - (4n + 3)^2</math> has a factor of 8, <math>1 - (4n + 3)^2</math> is a multiple of 8.</p>
5	$\frac{\theta}{2\pi - \theta} = \frac{1}{2.5}$ $2.5\theta = 2\pi - \theta$ $\theta = \frac{4\pi}{7}$
6 (a)	$k + (k - 7) = k - 2$ $k = 5$
6 (b)	$5x^2 - 2x - 3 = 0$ $(x - 1)(5x + 3) = 0$ $x = 1 \text{ or } x = -\frac{3}{5}$
7 (a)	The pricing appears to have tripled when it did not.
7 (b)	The y-axis should start from 0 to give a fairer comparison.
8	<p>Ratio of volume = <math>\left(\frac{1}{50}\right)^3</math></p> <p>Volume of model = <math>500\left(\frac{1}{50}\right)^3 \times 100^3 = 4 \times 10^4 \text{ cm}^3</math></p>
9	<p>Using cosine rule,</p> $5^2 = 3^2 + 4^2 - 2(3)(4)\cos \hat{A}BC$ $\angle ABC = 90^\circ$ <p><math>ABC</math> is a right-angled triangle <math>\Rightarrow AC</math> is the diameter (right-angle in semicircle)</p> <p>Area of circle = <math>\pi(2.5)^2 = 19.6</math> (3sf) <math>\text{cm}^2</math></p>

10	Difference = $\frac{467}{5\frac{5}{6}} - \frac{400}{6\frac{1}{3}} = 16.9$ (3sf) km/h
11 (a)	$4a^5b - 9ab^3 = ab(4a^4 - 9b^2)$ $= ab(2a^2 - 3b)(2a^2 + 3b)$
11 (b)	$3p^2q^2 - 2pq^2 + 6p - 4 = 3p(pq^2 + 2) - 2(pq^2 + 2)$ $= (pq^2 + 2)(3p - 2)$
12 (a)	1 worker would take 100 h to paint 10 classrooms. 1 worker would take 200 h to paint 20 classrooms. 5 workers would take <u>40 h</u> to paint 20 classrooms.
12 (b)	$d = ks$ $D = k(3s)$ $D = 3ks$ $D = 3d$ Percentage change = $\frac{3ks - ks}{ks} \times 100\% = 200\%$
13	Let the number of apples be $A$ . $75A + 95(A - 2) = 1000$ Apple = 7      Pears = 7 - 2      Total = 7 + 5 = 12
14 (a)	Median = 139.5 cm
14 (b)	Interquartile range = 26 cm
14 (c)	1. The mean height of the boys is higher at $143\frac{2}{9}$ cm than the girls' mean height at $137\frac{19}{22}$ cm. 2. The modal height of the girls is higher at 139 cm than the boys' modal height at 127 cm.
15	$9 = 3 - p + q$ .....(1) $12 = 12 + 2p + q$ .....(2)  (2) - (1): $3 = 9 + 3p$ $p = -2$ $q = 4$
16	Deposit = $0.2(97750 - 2000) = 19150$ Balance = $97750 - 2000 - 19150 = 76600$ Interest = $\frac{76600 \times 1.8 \times 5}{100} = 6894$ Total = $19150 + 76600 + 6894 = \$102644$



17	$\angle PQS = 90 - 38 = 52$ $\frac{1}{2}(15)(QS) \sin 52^\circ = \frac{1}{2}(12)(QS) \sin \angle QSR$ $\angle SQR = 80.1^\circ$ (1dp)
18	$y = \frac{5-x^2}{x^2+a}$ $y(x^2+a) = 5-x^2$ $x^2y+x^2 = 5-ay$ $x^2 = \frac{5-ay}{y+1}$ $x = \pm \sqrt{\frac{5-ay}{y+1}}$
19 (a)	$\angle BAE = \angle CDE$ (angles in the same segment) $\angle BEA = \angle CED$ (vertically opposite angles) Since triangle $ABE$ and triangle $DCE$ share 3 corresponding angles, the two triangles are similar.
19 (b)	$\frac{1}{2}(AD)(CD) = \frac{1}{2}(AD)(CD) \sin \angle ADC$ $\angle ADC = 90^\circ$ Since $ADC$ is a right-angled triangle, $AC$ is the diameter of the circle because of the circle property that the angle in a semicircle is always $90^\circ$ .
20	Third polygon's interior angle = $360 - \frac{4(180)}{6} - \frac{10(180)}{12}$ (sum of angles at a point) = 90 The polygon is a <u>square</u> .
21 (a)	$\angle BPC = 40$ (base angles of isosceles triangle) $\angle ABP = 40$ (alternate angles) Since $\angle BPC = \angle ABP$ , $BP$ is an angle bisector.
21 (b)	Let the vertical height of $\triangle BCP$ be $h$ . $\frac{\text{Area of } BCP}{\text{Area of } ABPD} = \frac{\frac{1}{2}(5)(h)}{8(h) - \frac{1}{2}(5)(h)}$ $= \frac{5}{11}$
22 (a)	$504 = 2^3 \times 3^2 \times 7$
22 (b)	$m = 3 \times 7^2 = 147$
22 (c)	HCF = $2 \times 3 \times 7$ $x = 2 \times 3 \times 7 \times p$ where $p = 5, 7, 11, 13, \dots$ Largest $x = 2 \times 3 \times 7 \times 11 = 462$

23 (a)	$(3k)^2 + (4k)^2 = 15^2$ $k = \pm \sqrt{\frac{225}{25}}$ $k = \pm 3$
23 (b)	(i) $\overline{PQ} = \begin{pmatrix} 3 \\ 15 \end{pmatrix} - \begin{pmatrix} -1 \\ 7 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$
	(ii) Angle = $\tan^{-1}\left(\frac{8}{4}\right) = 63.4^\circ$ (1dp)
	(iii) $\overline{PR} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} - \begin{pmatrix} -1 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ <p>Since <math>\overline{PR}</math> is not a scalar multiple of <math>\overline{PQ}</math>, the 2 vectors are not parallel which means that <math>R</math> does not lie on the line <math>PQ</math>.</p>