Name : ___________________________ (   )   Class : Sec 3 / ____   Date : _______

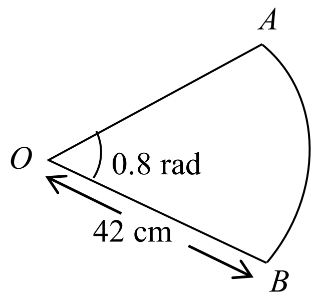
Chapter 10.4 Formulae in Radian Measure

Question 1:

The diagram shows a blade from the windmill. The blade *OAB* has a radius of 42 cm and subtends an angle of 0.8 radian at the centre. Calculate the

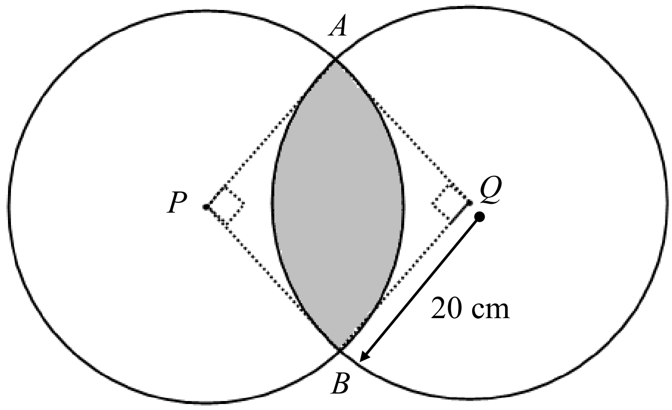
(a) area of the blade,

(b) perimeter of the blade.



Question 2:

Two circles with centre at *P* and *Q* respectively overlap at points *A* and *B*. Both circles have a radius of 20 cm. Find the area of shaded region.



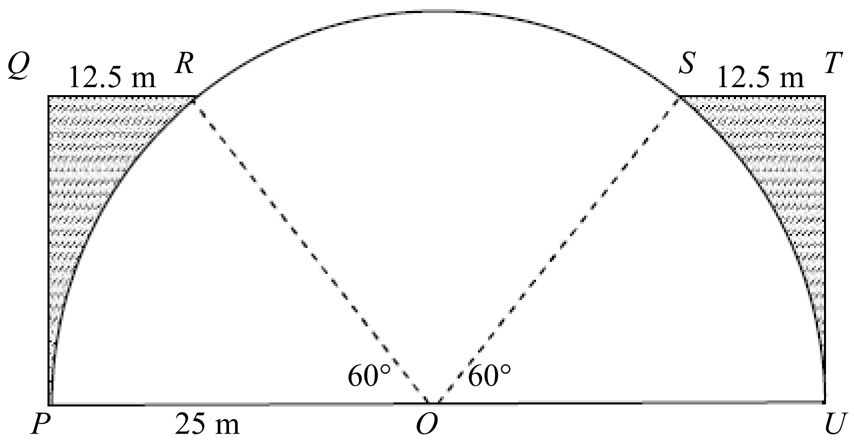
Question 3:

The diagram below shows a semi-circular shaped swimming pool with shaded parts *PQR* and *STU* being the sitting areas. Given that $OP = 25$ m and $\angle POR = 60°$. Find

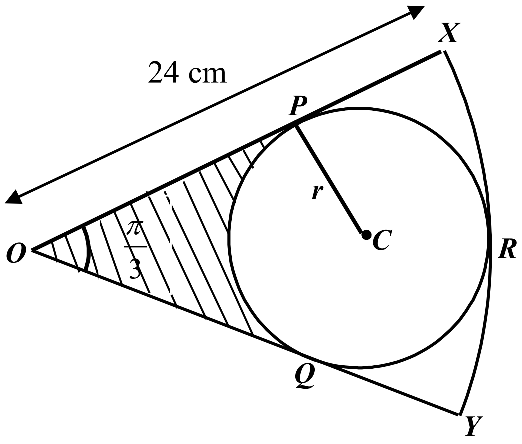
(a) the arc length *RS*,

(b) the perimeter of the sitting areas,

(c) calculate the total shaded area.



Question 4:



In the figure, $OXRY$ is a sector of a circle with centre O and radius 24 cm. The circle PQR with centre C and radius r cm is inscribed in the sector.

Given that $\angle POQ = \frac{\pi}{3}$ rad. OX and OY are tangents to the circle PQR at P and Q respectively.

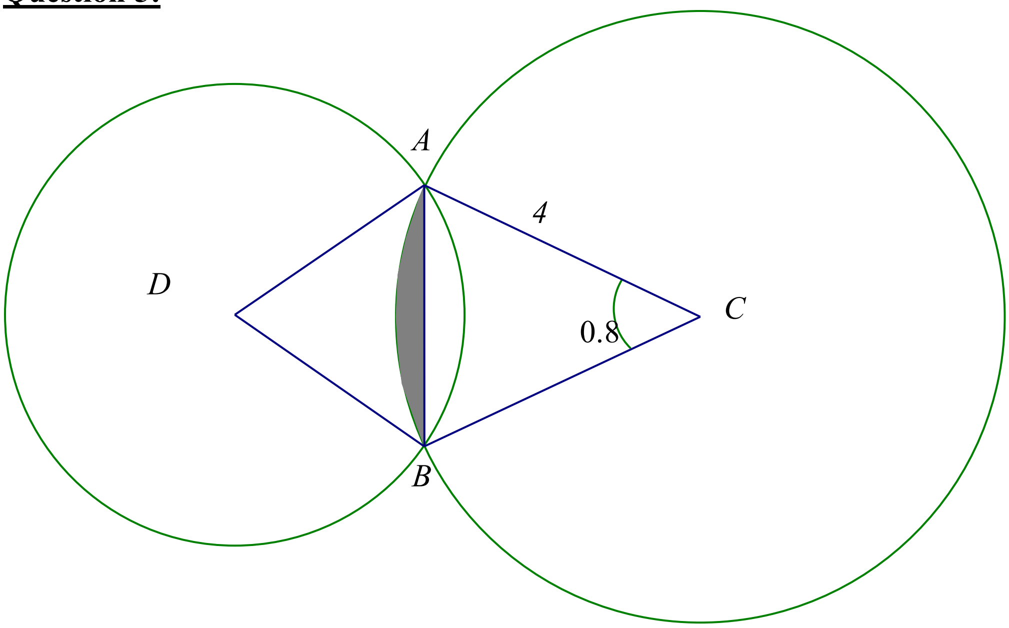
(a) Explain briefly why angle OPC is a right angle.

(b) Show that $OC = 2r$.

(c) Hence, find the value of r and the length of OP.

(d) Find the area of the shaded region.

Question 5:



(a) The points A and B lie on a circle, centre C. $AC = BC = 4$ cm and $\angle ACB = 0.87$ radians.

  (i) Find the length of the arc AB.

  (ii) Calculate the area of the sector CAB.

  (iii) Calculate the area of the shaded segment.

(b) A second circle, centre D, cuts the first circle at A and B. The radius of the second circle is 3 cm and $\angle CBD = 2.095$ radians.

  (i) Calculate CD.

  (ii) Calculate, in radians, the angle ADB.

  (iii) Calculate the perimeter of the sector ADB.

Question 6:

In the figure, *OAB* is a sector of a circle with centre *O*. A semi-circle with radius *QB* touching *OA* at *P* is drawn in the sector. If angle $POQ = 60°$, and OQ = 10 cm, find

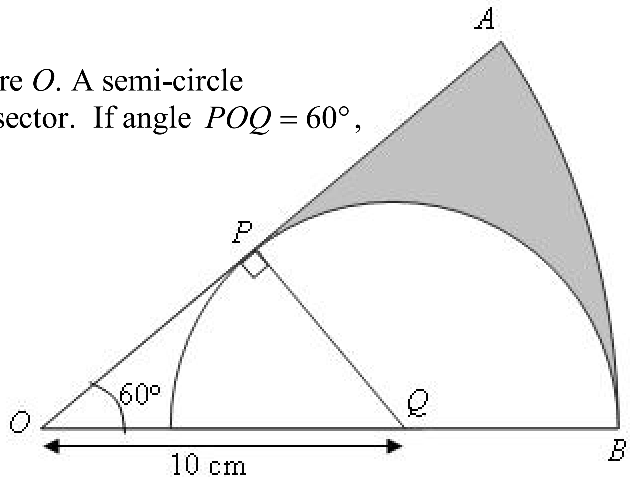
(i) the radius of the semi-circle, *PQ*,

(ii) $\angle PQB$,

(iii) the area of triangle *PQO*,

(iv) the area of the sector *PQB*,

(v) the area of the shaded region.



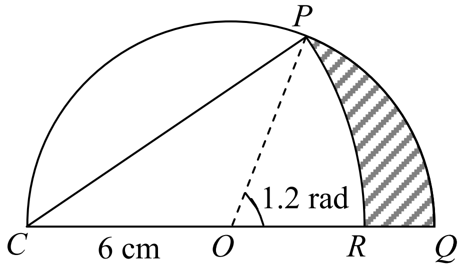
Question 7:

In the diagram below, *OCPQ* is a semicircle of radius 6 cm, with centre *O*. *CPR* is a sector of the circle with centre *C*. The angle *POR* is 1.2 radians.

(i) Show that the length of *PC* is approximately 9.90 cm.

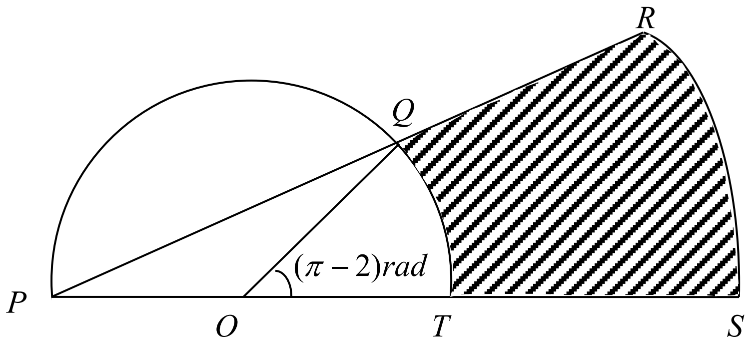
(ii) Find the perimeter of the shaded region.

(iii) Find the area of the shaded region.



Question 8:

The figure below shows a semicircle *PQT* with centre *O*. *PRS* is a sector of a circle with centre *P*. *T* is the midpoint of *PS*. The length of *OP* = 4 cm and $\angle QOT = (\pi - 2)$ radians.



Calculate

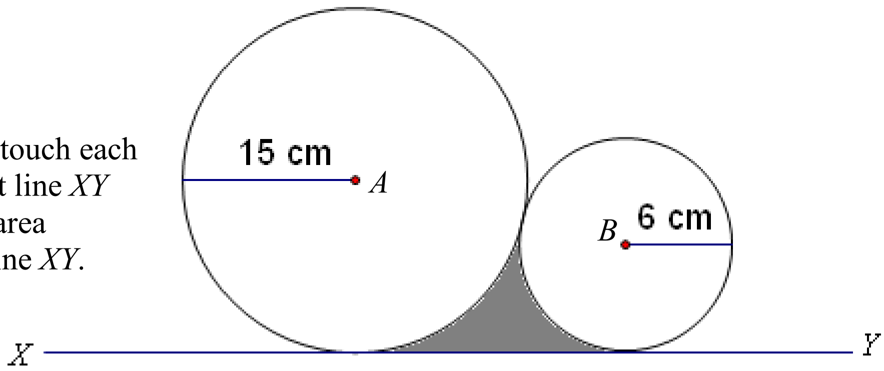
(a) the length of the arc *PQ* in cm,

(b) the perimeter, in cm, of the shaded region,

(c) the area, in cm^2, of the shaded region.

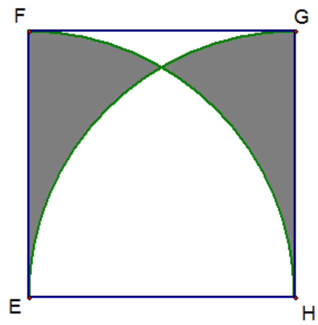
Question 9:

Two coins of radii 15 cm and 6 cm touch each other externally and lie on a straight line *XY* as shown in the diagram. Find the area enclosed by the two coins and the line *XY*.



Question 10:

The diagram above shows a square *EFGH* of side 8 cm.
Arc *EG* and Arc *FH* are drawn with a radius of 8 cm from centers *H* and *E* respectively. Find the area of the shaded region.



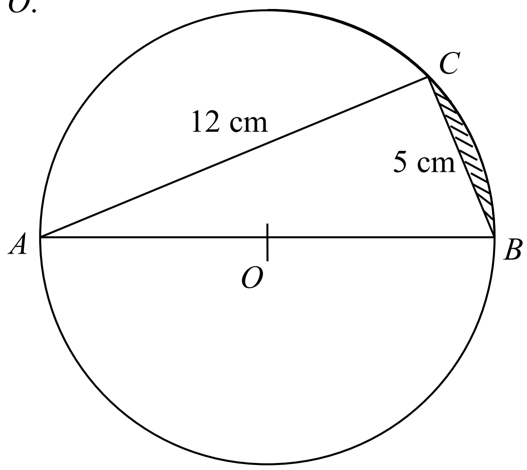
Question 11:

In the diagram, *AB* is the diameter of a circle with centre *O*. *AC* and *BC* are two chords of length 12 cm and 5 cm respectively. Calculate

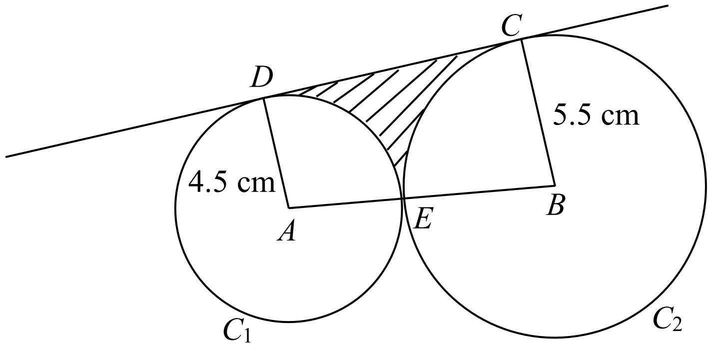
(a) the radius of the circle,

(b) $\angle CAB$ in radians,

(c) the area of the shaded region.



Question 12:



The diagram shows two circles C_1 and C_2, touching at *E*. Circle C_1 has a radius of 4.5 cm and centre *A*, circle C_2 has a radius of 5.5 cm and centre *B*. A tangent touches both circles at the points *D* and *C* respectively.

(a) Explain why *AD* is parallel to *BC*.

(b) State the special name given to the quadrilateral *ABCD*.

(c) Calculate

 (i) the length of *CD*,

 (ii) $\angle DAB$ in radians,

 (iii) the perimeter of the shaded region,

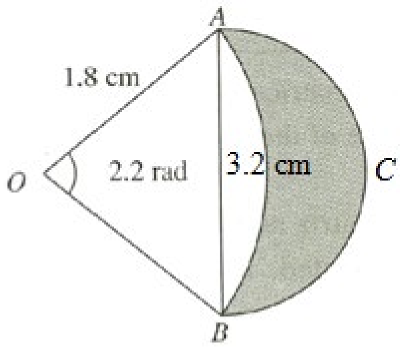
 (iv) the area of the shaded region.

Question 13:

The figure shows a sector OAB with centre O overlapping a semicircle ACB of diameter AB. Given that $OA = 1.8$ cm, $AB = 3.2$ cm and $\angle AOB = 2.2$ radians, calculate

(a) the perimeter of the shaded region,

(b) the area of the shaded crescent.



Question 14:

The diagram shows an arc AB of a circle with center O and another arc CB of a circle with center D. Given that $OC = 6$ cm, $OD = 9$ cm, $\angle OCD = 0.9$ rad and $\angle COD = 1.692$ rad, find

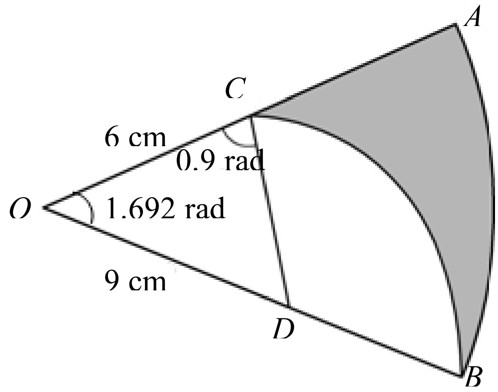
(a) the length of CD,

(b) the length of AC,

(c) the length of arc AB,

(d) the perimeter of the shaded region,

(e) the area of the shaded region.

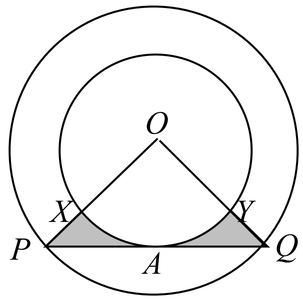


Question 15:

The diagram shows two concentric circles with centre O. The radii of the circles are 16 cm and 25 cm respectively. PAQ is a tangent to the inner circle at A. Calculate

(a) $\angle POQ$ in radians,

(b) the area of the shaded region.



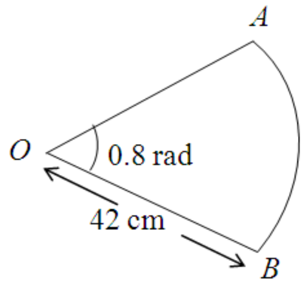
Answer Key

1.	(a)	705.6 cm^2	(b)	117.6 cm		
2.	228 cm^2					
3.	(a)	26.2 m	(b)	121 m	(c)	157 m^2
4.	(a)	Tangent is perpendicular to radius			(c)	$OP = 13.9$ cm
	(d)	43.8 cm^2				
5.	(a)(i)	3.48 cm	(ii)	6.96 cm^2	(iii)	0.845 cm^2
	(b)(i)	6.08 cm	(ii)	1.22 rad	(iii)	9.67 cm
6.	(i)	8.66 cm	(ii)	150°	(iii)	21.7 cm^2
	(iv)	98.2 cm^2	(ii)	62.5 cm^2		
7.	(ii)	15.2 cm	(iii)	8.95 cm^2		
8.	(a)	8 cm	(b)	31.0 cm	(c)	56.7 cm^2
9.	36.1 cm^2					
10.	21.9 cm^2					
11.	(a)	6.5 cm	(b)	0.395 rad	(c)	1.68 cm^2
12.	(b)	Trapezium	(c)(i)	9.95 cm	(ii)	1.67 rad
	(iii)	25.6 cm	(iv)	10.6 cm^2		
13.	(a)	8.99 cm	(b)	1.77 cm^2		
14.	(a)	11.4 cm	(b)	14.4 cm	(c)	34.5 cm
	(d)	78.5 cm	(e)	157 cm^2		
15.	(a)	1.75 rad	(b)	83.0 cm^2		

Solutions

1(a) Area of blade $= \frac{1}{2} \times 42^2 \times 0.8$

$= 705.6 \text{ cm}^2$



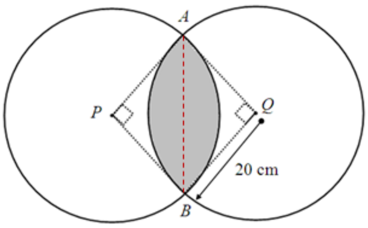
(b) Perimeter of the blade $= OA + OB + AB$

$= 42 + 42 + 42(0.8)$

$= 117.6 \text{ cm}$

2 Area of quadrant $PAB = \frac{1}{4} \times \pi \times 20^2$

$= 100\pi \text{ cm}^2$



Area of $\Delta PAB = \frac{1}{2} \times 20 \times 20$

$= 200 \text{ cm}^2$

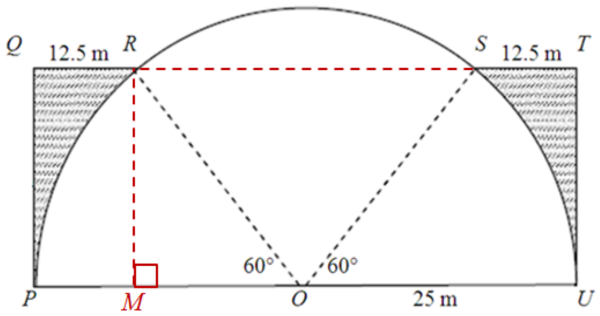
Area of shaded area $= (100\pi - 200) \times 2$

$= 228 \text{ cm}^2 \text{ (3 sf)}$

3(a) Arc length $RS = \frac{60°}{360°} \times 2 \times \pi \times 25$

$= 26.1799$

$= 26.2 \text{ m (3 sf)}$



(b) In ΔOMR,

$\sin 60° = \frac{MR}{25}$

$MR = 25\sin 60°$

$= 21.6506 \text{ m}$

$QP = TU = MR = 21.6506 \text{ m}$

Perimeter of sitting areas

$= 2 \times [PQ + QR + \text{arc } PR]$

$= 2 \times [PQ + QR + \text{arc } RS]$

$= 2 \times [21.6506 + 12.5 + 26.1799]$

$= 121 \text{ m (3 sf)}$

(c) Area of rectangle $PQTU = 21.6506 \times (25 + 25)$

$= 1082.53 \text{ m}^2$

Area of sector $OPQ = \dfrac{60°}{360°} \times \pi \times 25^2$

$= 327.2492 \text{ m}^2$

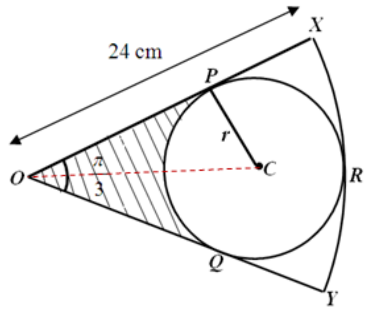
Area of sector $\Delta ORS = \dfrac{1}{2} \times 25 \times 25 \times \sin 60°$

$= 270.6329 \text{ m}^2$

Total shaded area $= 1082.53 - 2(327.2492) - 270.6329$

$= 157 \text{ m}^2$ (3 sf)

4(a) Given OX is tangent to the circle at P and CP is the radius of the circle.
Then, OX is perpendicular to CP
Hence, angle OPC is a right angle.



(b) In ΔOPC,

$\sin \dfrac{\pi}{6} = \dfrac{r}{OC}$

$OC = \dfrac{r}{\sin \dfrac{\pi}{6}}$

$= 2r$ (shown)

(c) $OR = OC + CR$

$24 = 2r + r$

$3r = 24$

$r = 8 \text{ cm}$

In ΔOPC,

$OP = \sqrt{16^2 - 8^2}$ (Pythagoras' Thm)

$= \sqrt{192}$

$= 13.9 \text{ cm}$ (3sf)

(d) Area of $\Delta OPC = \dfrac{1}{2} \times OP \times PC$

$= \dfrac{1}{2} \times \sqrt{192} \times 8$

$= 55.4256 \text{cm}^2$

$\angle OCP = \pi - \frac{\pi}{2} - \frac{\pi}{6}$ ($\angle$ sum of Δ)

$= \frac{\pi}{3}$ rad

Area of sector $= \frac{1}{2} \times 8^2 \times \frac{\pi}{3}$

$= 33.5103\,\text{cm}^2$

Area of shaded region

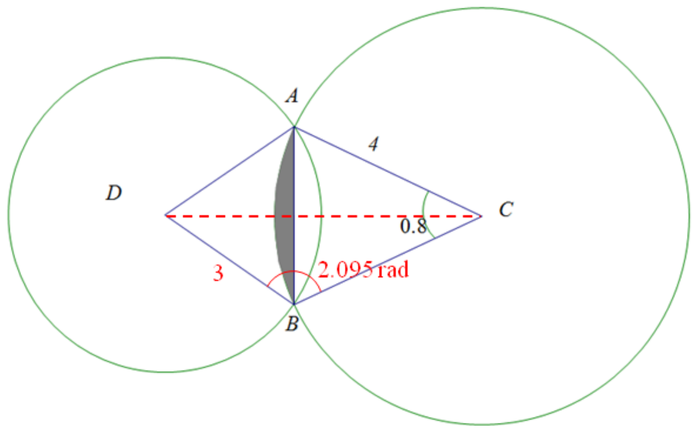
$= 2 \times [55.4256 - 33.5103]$

$= 43.8\,\text{cm}^2$ (3 sf)

5(a) (i) Length of arc AB

$= 4 \times 0.87$

$= 3.48$ cm



(ii) Area of sector CAB

$= \frac{1}{2} \times 4^2 \times 0.87$

$= 6.96\,\text{cm}^2$

(iii) Area of ΔABC

$= \frac{1}{2} \times 4^2 \times \sin 0.87$

$= 6.11463\,\text{cm}^2$

Area of shaded region

$= 6.96 - 6.11463$

$= 0.84537$

$= 0.845\,\text{cm}^2$ (3sf)

(b) (i) In ΔCBD,

$CD = \sqrt{3^2 + 4^2 - 2(3)(4)\cos 2.095}$

$= 6.08380$

$= 6.08$ cm (3sf)

(ii) $DACB$ is a kite.

$\angle ADB = 2\pi - 2(2.095) - 0.87$ ($\angle$ sum of quad)

$= 1.22318$

$= 1.22$ rad (3 sf)

(iii) Perimeter of sector

$= 3 + 3 + 3(1.22318)$

$= 9.66954$

$= 9.67\,\text{cm}\ (3\text{ sf})$

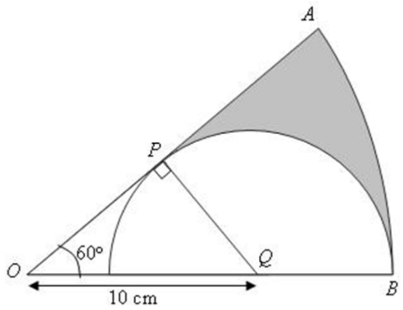
6(i) In ΔPQR,

$$\sin 60° = \frac{PQ}{10}$$

$$10\sin 60° = PQ$$

$$PQ = 8.66025$$

$$= 8.66\,\text{cm}\ (3\text{ sf})$$



(ii) $\angle PQB = 90° + 60°$ (ext $\angle$s of Δ)

$= 150°$

(iii) Area o f $\Delta PQO = \frac{1}{2} \times 10 \times 8.66025 \times \sin 30°$

$= 21.65062$

$= 21.7\ \text{cm}^2\ (3\text{ sf})$

(iv) Area of sector $PQB = \frac{150°}{360°} \times \pi \times (8.66025)^2$

$= 98.17468$

$= 98.2\ \text{cm}^2\ (3\text{ sf})$

(v) Area of sector $OAB = \frac{60°}{360°} \times \pi \times (10 + 8.66025)^2$

$= 182.31968\ cm^2$

Area of shaded region

$= 182.31968 - 21.65062 - 98.17468$

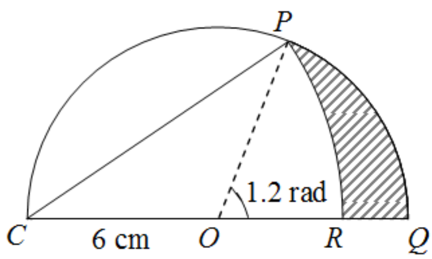
$= 62.5\ \text{cm}^2\ (3\text{ sf})$

7(i) In ΔCOP,

$$PC = \sqrt{6^2 + 6^2 - 2(6)(6)\cos(\pi - 1.2)}$$

$= 9.90403$

$= 9.90$ cm (3sf) (shown)



(ii) Arc length $PQ = 6 \times 1.2$
$= 7.2\,\text{cm}$

Since ΔCOP is isosceles,
$\angle PCO = 1.2 \div 2$ (ext $\angle$s of Δ)
$= 0.6$ rad

Arc length $PR = 9.90403 \times 0.6$
$= 5.94242\,\text{cm}$

$RQ = CQ - CR$
$= 12 - 9.90403$
$= 2.09597\,\text{cm}$

Perimeter of shaded region
$= \text{arc } PQ + \text{arc } PR + RQ$
$= 7.2 + 5.94242 + 2.09597$
$= 15.2\,\text{cm}$ (3sf)

(iii) Area of sector $OPR = \frac{1}{2} \times 6^2 \times 1.2$
$= 21.6\ \text{cm}^2$

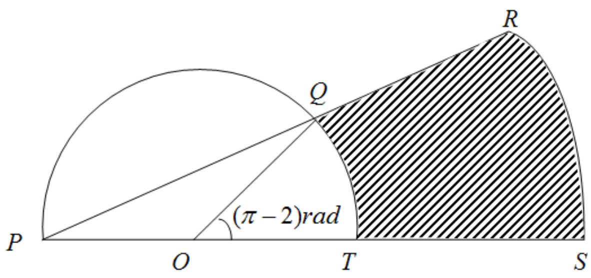
Area of segment PR
$= \left[\frac{1}{2} \times (9.90403)^2 \times 0.6\right] - \left[\frac{1}{2} \times (9.90403)^2 \times \sin 0.6\right]$
$= 1.7341\ \text{cm}^2$

Area of $\Delta OPR = \frac{1}{2} \times 6 \times (9.90403 - 6) \times \sin 1.2$
$= 10.9161\ \text{cm}^2$

Area of shaded region
= Area of sector OPQ – Area of segment PR – Area of ΔOPR
$= 21.6 - 1.7341 - 10.9161$
$= 8.95\ \text{cm}^2$ (3sf)

8(a) $\angle POQ = \pi - (\pi - 2)$ (adj $\angle$s on a str line)
$= 2$ rad

Length of arc $PQ = 4 \times 2$
$= 8$ cm



(b) Arc length $QT = 4 \times (\pi - 2)$
$= 4.568$ cm

Since T is the mid-point of PS, $TS = 8$ cm

Since ΔOPQ is isosceles,
$\angle QPT = \angle QOT \div 2$ (ext $\angle$s of Δ)
$= \dfrac{\pi - 2}{2}$ rad

Arc length $RS = 16 \times \dfrac{\pi - 2}{2}$
$= 8\pi - 16$
$= 9.136$ cm

In ΔOPQ,
$PQ = \sqrt{4^2 + 4^2 - 2(4)(4)\cos 2}$
$= 6.73177$ cm

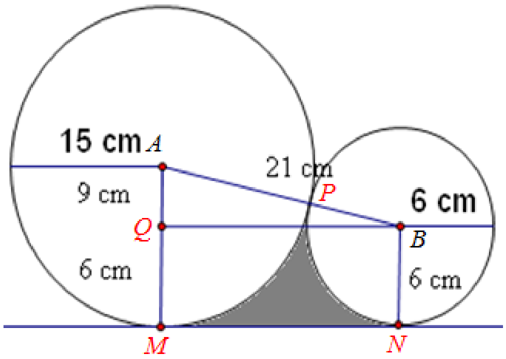
$QR = 16 - 6.73177$
$= 9.26823$ cm

Perimeter of shaded region
$= \text{arc } QT + \text{arc } RS + TS + QR$
$= 4.568 + 9.136 + 8 + 9.26823$
$= 31.0$ cm (3 sf)

(c) Area of shaded region
= Area of sector PRS – Area of sector OQT – Area of ΔOPQ
$$= \left[\frac{1}{2}(16)^2(\frac{\pi - 2}{2})\right] - \left[\frac{1}{2}(4)^2(\pi - 2)\right] - \left[\frac{1}{2}(4)^2 \sin 2\right]$$
$= 56.7 \text{ cm}^2$ (3 sf)

9 In ΔABE,

$$EB = \sqrt{(15+6)^2 - 9^2} \quad \text{(Pythagoras' Thm)}$$
$$= \sqrt{360} \text{ cm}$$



$$\text{Area of trapesizum} = \frac{1}{2} \times (6+15) \times \sqrt{360}$$
$$= 199.2235 \text{ cm}^2$$

In ΔABE,

$$\cos \angle EAB = \frac{9}{21}$$
$$\angle EAB = 64.6231°$$

$$\text{Area of sector } DAF = \frac{64.6231°}{360°} \times \pi \times 15^2$$
$$= 126.8871 \text{ cm}^2$$

In ΔABE,

$$\angle ABE = 180° - 90° - 64.6231° \quad (\angle \text{ sum of } \Delta)$$
$$= 25.3769°$$
$$\angle FBC = 90° + 25.3769°$$
$$= 115.3769°$$

$$\text{Area of sector } FBC = \frac{115.3769°}{360°} \times \pi \times 6^2$$
$$= 36.2467 \text{ cm}^2$$

Area of shaded region

$= \text{Area of tapezium} - \text{Area of sector } DAF - \text{Area of sector } FBC$

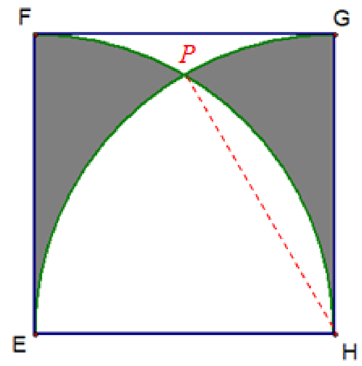
$= 199.2235 - 126.8871 - 36.2467$

$= 36.1 \text{ cm}^2 \text{ (3 sf)}$

10. $\angle HEC = 60°$ (equilateral Δ)

$\angle CEF = 90° - 60°$ (comp $\angle$s)

$= 30°$



$$\text{Area of sector } CEF = \frac{30°}{360°} \times \pi (8)^2$$
$$= 16.7552 \text{ cm}^2$$

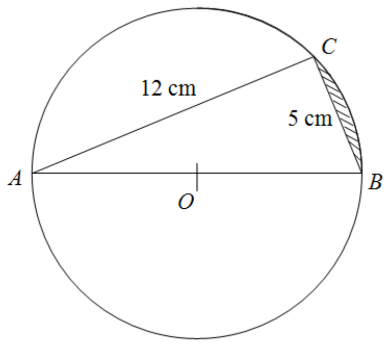
$$\text{Area of segment } CE = \frac{60°}{360°} \times \pi (8)^2 - \frac{1}{2}(8)^2 \sin 60°$$
$$= 5.7975 \text{ cm}^2$$

$$\text{Area of shaded region} = 2 \times (16.7552 - 5.7975)$$
$$= 21.9 \text{ cm}^2 \quad (3\text{ sf})$$

11(a) In ΔABC,

$AB = \sqrt{12^2 + 5^2}$ (Pythagoras' Thm)

$= 13$ cm



Radius of circle $= 13 \div 2$

$= 6.5$ cm

(b) In ΔABC,

$$\tan \angle CAB = \frac{5}{12}$$
$$\angle CAB = 0.3948$$
$$= 0.395 \text{ rad (3 sf)}$$

(c) $\angle COB = 2(\angle CAB)$

$= 2(0.39479)$

$= 0.78958$ rad

$$\text{Area of shaded region} = \text{Area of sector } OCB - \text{Area of } \Delta OCB$$
$$= \frac{1}{2}(6.5)^2(0.78985) - \frac{1}{2}(6.5)^2 \sin 0.78958$$
$$= 1.67991$$
$$= 1.68 \text{ cm}^2 \quad (3\text{sf})$$

12(a) Since DC is tangent to both circles,

$$\angle ADC = \angle BCD = 90° \text{ (tangent} \perp \text{radius)}$$

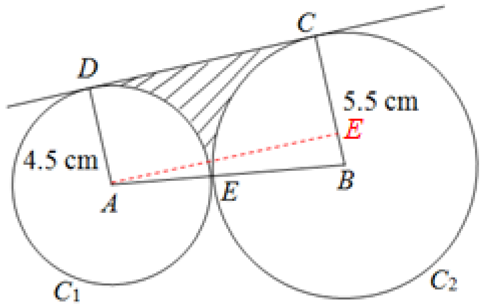
Then, $\angle ADC + \angle BCD = 180°$

$\therefore$ $AD // BC$ because the sum of these interior angles $\angle ADC$ and $\angle BCD$ is 180°.

(b) $ABCD$ is a trapezium.

(c) (i) In ΔABE,

$$CD = \sqrt{(AE)^2 - (EB)^2} \text{ (Pythagoras' Thm)}$$
$$= \sqrt{(4.5+5.5)^2 - (5.5-4.5)^2}$$
$$= 9.94987$$
$$= 9.95 \text{ cm (3sf)}$$



(ii) In ΔABE,

$$\sin \angle EAB = \frac{1}{(5.5+4.5)}$$
$$\angle EAB = 0.10017 \text{ rad}$$
$$\angle DAB = \frac{\pi}{2} + 0.10017$$
$$= 1.67096$$
$$= 1.67 \text{ rad (3 sf)}$$

(iii) Perimeter of shaded region

$$= CD + \text{arc } DE + \text{arc } CE$$
$$= 9.94987 + (4.5)1.67096 + (5.5)(\pi - 1.67096)$$
$$= 25.55767$$
$$= 25.6 \text{ cm (3sf)}$$

(iv) Area of shaded region

$$= \text{Area of trapezium} - \text{Area of 2 sectors}$$
$$= \frac{1}{2}(4.5+5.5)\left(\sqrt{99}\right)$$
$$-\frac{1}{2}(4.5)^2(1.67096) - \frac{1}{2}(5.5)^2(\pi - 1.67096)$$
$$= 49.74937 - 16.91847 - 22.24332$$
$$= 10.58758$$
$$= 10.6 \text{ cm}^2 \text{ (3sf)}$$

13(a) Circumference of semi-circle

$= 0.5 \times \pi \times 3.2$

$= 5.02655$ cm

Length of arc *AB*

$= 1.8 \times 2.2$

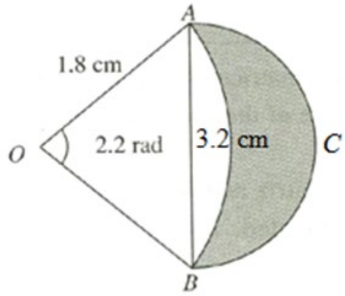
$= 3.96$ cm

Perimeter of shaded region

$= 5.02655 + 3.96$

$= 8.98655$

$= 8.99 \text{cm}$ (3sf)



(b) Area of semi-circle *ACB*

$= 0.5 \times \pi \times (3.2 \div 2)^2$

$= 4.02124 \text{ cm}^2$

Area of segment *AB*

= Area of sector *OAB* – Area of Δ*OAB*

$= 0.5 \times (1.8)^2 \times (2.2 - \sin 2.2)$

$= 2.25424 \text{cm}$

Area of shaded crescent

$= 4.02124 - 2.25424$

$= 1.76700$

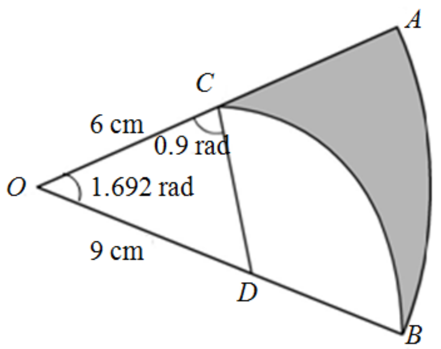
$= 1.77 \text{cm}^2$ (3sf)

14(a) In Δ*COD*,

$$CD = \sqrt{6^2 + 9^2 - 2(6)(9)\cos 1.692}$$

$= 11.404296$

$= 11.4$ cm (3sf)



(b) $OB = OD + DB$

$= 9 + 11.404296$

$= 20.404296 \text{cm}$

$AC = OA - OC$

$= 20.404296 - 6$

$= 14.404296$

$= 14.4 \text{cm}$ (3sf)

(c) Length of arc AB
$= 20.404296 \times 1.692$
$= 34.52409$
$= 34.5\,\text{cm}$ (3sf)

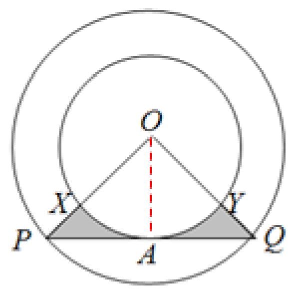
(d) In ΔCOD,
$\angle CDB = 1.692 + 0.9$ (ext $\angle$ of Δ)
$= 2.592\,\text{rad}$

Length of arc CB
$= 11.404296 \times 2.592$
$= 29.55994\,\text{cm}$

Perimeter of shaded region
$= AC + arc\ AB + arc\ CB$
$= 14.404296 + 34.52409 + 29.55994$
$= 78.48832$
$= 78.5\,\text{cm}$ (3sf)

(e) Area of shaded region
= Area of sector OAB − Area of sector DCB − Area of ΔCOD
$= 0.5(20.404296)^2(1.692) - 0.5(11.404296)^2(2.592)$
$- 0.5(6)(9)\sin 1.692$
$= 352.21966 - 168.55513 - 26.80192$
$= 156.86261$
$= 157\,\text{cm}^2$ (3sf)

15(a) Given PQ is tangent at A, then $\angle OAP = 90°$
$OA = OX = OY = 16\,\text{cm}$
$OP = OQ = 25\,\text{cm}$



In ΔOAP,
$\cos\angle POA = \dfrac{16}{25}$
$\angle POA = 0.876298$

$\angle POQ = 2(0.876298)$
$= 1.752596$
$= 1.75\,\text{rad}$ (3sf)

(b) Area of ΔPOQ

$= 0.5(25)(25)\sin 1.752596$

$= 307.34996\,\text{cm}^2$

Area of minor sector XOY

$= 0.5(16)^2(1.752596)$

$= 224.33229\,\text{cm}^2$

Area of shaded region

$= 307.34996 - 224.33229$

$= 83.01767$

$= 83.0\,\text{cm}^2$ (3sf)