

Topical Worksheet: Quadratic Equations
Secondary 3 Mathematics

BASIC

1. Solve the following equations by the factorisation method.
 - (a) $x^2 + 14x + 48 = 0$
 - (b) $x^2 + 7x - 18 = 0$
 - (c) $2x^2 + x - 28 = 0$
 - (d) $4x^2 + 16x + 15 = 0$
 - (e) $6x^2 - 23x + 7 = 0$
 - (f) $9x^2 - 18x - 16 = 0$
 - (g) $12x^2 - 13x - 35 = 0$
 - (h) $15x^2 - 19x + 6 = 0$
 - (i) $3x^2 - 48 = 0$
 - (j) $100 - 4x^2 = 0$

2. (a) Draw, on separate graph papers, the graph of
 - (i) $y = x^2 + x - 6$ for $-4 \leq x \leq 4$
 - (ii) $y = x^2 - 8x + 12$ for $0 \leq x \leq 8$
 - (iii) $y = x^2 + 4x + 2$ for $-4 \leq x \leq 2$
 - (iv) $y = x^2 - 5x + 3$ for $-2 \leq x \leq 6$
 - (v) $y = 2x^3 - 3x - 6$ for $-3 \leq x \leq 3$
 - (vi) $3x^2 + 2x - 4$ for $-3 \leq x \leq 3$

(b) Hence, solve each of the following equations graphically.

 - (i) $y = x^2 + x - 6 = 0$
 - (ii) $y = x^2 - 8x + 12 = 0$
 - (iii) $y = x^2 + 4x + 2 = 0$
 - (iv) $y = x^2 - 5x + 3 = 0$
 - (v) $y = 2x^3 - 3x - 6 = 0$
 - (vi) $3x^2 + 2x - 4 = 0$

3. Find the possible values of b if each of the following expressions is a perfect square.
 - (a) $x^2 + bx + 1$
 - (b) $x^2 + bx + 25$
 - (c) $x^2 + 2bx + 81$
 - (d) $x^2 + 5bx + 100$
 - (e) $x^2 + (b + 3)x + 49$
 - (f) $x^2 + (7 - b)x + 144$

4. Solve the following equations by completing the square method, giving your answers correct to 3 significant figures.
 - (a) $x^2 + 5x + 4 = 0$
 - (b) $x^2 - 8x + 16 = 0$
 - (c) $x^2 + 5x + 3 = 0$
 - (d) $x^2 - 7x + 9 = 0$
 - (e) $x^2 + 9x - 5 = 0$
 - (f) $x^2 + 4x - 1 = 0$
 - (g) $x^2 - 3x - 2 = 0$
 - (h) $x^2 - 6x - 5 = 0$

5. Solve the following equations by using the quadratic formula, giving your answers correct to 3 significant figures.
 - (a) $x^2 + 7x + 12 = 0$
 - (b) $2x^2 + 6x - 3 = 0$
 - (c) $x^2 - 6x + 6 = 0$
 - (d) $3x^2 - 4x - 2 = 0$
 - (e) $-3x^2 + 7x + 5 = 0$
 - (f) $-x^2 + x + 10 = 0$
 - (g) $-4x^2 - 5x + 7 = 0$
 - (h) $5x^2 - 17 = 0$

6. Solve the following equations.

(a) $\frac{x}{3} = \frac{27}{x}$

(b) $\frac{x}{x-4} = x - 3$

(c) $\frac{x+2}{3} = \frac{2}{x+7}$

(d) $\frac{x+6}{x+8} = \frac{x-2}{x-9}$

(e) $\frac{12}{x} - 4 = \frac{x}{2}$

(f) $\frac{1}{x-12} + \frac{3}{x-10} = \frac{14}{15}$

(g) $\frac{1}{x+2} - \frac{1}{x-5} = \frac{7}{10}$

(h) $\frac{3}{x-4} - \frac{2}{x+1} = \frac{2}{5}$

7. The difference between 2 positive numbers is 4. The sum of the squares of the 2 numbers is 106. Let the smaller number be x .

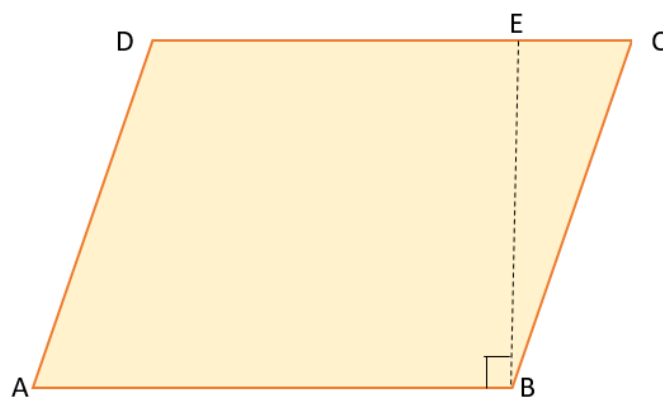
- Express the greater number in terms of x .
- Form an equation in x and solve it.
- Hence, find the sum of the 2 numbers.

8. In a two-digit number, the ones digit is twice the tens digit. The number is twice the product of the digits. Let the tens digit be y .

- Express the ones digit in terms of y .
- Form an equation in y and solve it.
- Hence, write down the number.

9. In the figure, ABCD is parallelogram. $AB = (2x + 5)$ cm, $BE = (x - 2)$ cm, BE is perpendicular to AB and the area of ABCD is 90cm^2 .

- Form an equation in x and solve it.
- Hence, find the lengths of AB and BE. Give your answers correct to 3 significant figures.



10. A solid cuboid is $(4x - 1)$ cm long, $2x$ cm wide and $(2x + 1)$ cm high.
- Form an equation in x if the total surface area of the cuboid is 240cm^2 .
 - Solve the equation in (a).
 - Hence, find the volume of the cuboid. Give your answer correct to the nearest cm^3 .
11. Solve the following equations by the factorisation method.
- $x^2 - 6x = 4x - 21$
 - $x(2x + 3) = 6(3 - x)$
 - $4x(x - 4) = 7(x - 4)$
 - $(5x - 3)6x = 9(3 - 5x)$
 - $(x + 12)(x - 3) = 9x$
 - $(2x - 3)(x + 3) = 4x - 3$
 - $(4 - 5x)^2 + 5(4 - 5x) + 6 = 0$
 - $(x - 1)^2 - 7(1 - x) - 18 = 0$
 - $4(2x + 1)^2 + 4(2x + 1) - 63 = 0$
12. (a) (i) Draw the graph of $y = (2x - 3)(3x + 7)$ for $-3 \leq x \leq 3$.
(ii) Hence, solve the equation $6x^2 + 5x - 21 = 0$ graphically.
- (b) (i) Draw the graph of $y = (2x + 5)(4 - x)$ for $-4 \leq x \leq 5$.
(ii) Hence, solve the equation $-2x^2 + 3x + 20 = 0$ graphically.
13. Solve the following equations by the completing the square method, giving your answers correct to 3 significant figures.
- $x^2 + 4x = 36$
 - $x^2 - 30 = 14x$
 - $(x + 3)(x + 2) = 9$
 - $(x - 4)(x - 1) = 2x$
 - $(x - 2)^2 = 5x - 2$
 - $5x^2 + 4x - 10 = 4x^2 - 3x - 4$
 - $(2x + 5)^2 = 3x(x - 3)$
 - $(1 - x)(2 - 3x) = (2x + 1)(x + 1)$
14. Solve the following equations by using the quadratic formula, giving your answers correct to 3 significant figures.
- $4x + x^2 + 1 = 0$
 - $5 - 9x + x^2 = 0$
 - $(2x - 3)(4x + 7) = 2$
 - $(x + 6)(5 - 2x) = 2(x + 1)$
 - $(x - 3)^2 = x$
 - $(x + 5)^2 = 3x(x - 6)$
 - $(x + 1)(2x + 3) = (3x + 5)(5x + 7)$
 - $(2 - 5x)(3 + x) = (2x + 5)^2$
15. Solve the following equations.
- $\frac{8-x}{x+7} + \frac{x+4}{3+x} = 4$
 - $\frac{2x+3}{x-3} - \frac{11-x}{x-1} = 5$
 - $\frac{3-x}{x-2} - \frac{3x+1}{x+2} = \frac{2}{3}$
 - $\frac{2+x}{4-x} + \frac{2-3x}{x+4} = \frac{4}{5}$
 - $\frac{5}{x+1} - \frac{x+10}{x^2-x-2} = 15$
 - $\frac{7}{x^2-3x-4} - \frac{x+5}{4-x} = 6$
 - $\frac{5}{x-5} - \frac{4}{x-2} = \frac{3}{x-3}$
 - $\frac{20}{x+8} + \frac{1}{x-24} = \frac{4}{x+2}$
16. (a) Express, as a single power of 2,
- $4^{\frac{1}{x}}$,
 - $\sqrt[3]{16}$.

(b) Use your answers in (a) to solve $2^{\frac{1}{x+1}} \times 4^{\frac{1}{x}} = \sqrt[3]{16}$.

17. The general term of a sequence is $T_n = n^2 + an + b$.

(a) Find the values of a and b if both the 4th term and the 6th term are 0.

(b) Hence, find the possible values of p if the p^{th} term is 8.

18. The slant height, base diameter and height of a solid cone are $3(x + 3)$ cm, $2(x + 1)$ cm and $4(x - 2)$ cm respectively.

(a) (i) Show that an equation in x is $2x^2 - 29x - 4 = 0$.

(ii) Solve the equation.

(b) Hence, calculate

(i) the volume of the cone correct to the nearest cm^3 ,

(ii) the total surface area of the cone correct to the nearest cm^2 .

19. A piece of wire is bent into the shape shown in the figure. ABC is a semicircle of radius 10 cm, $AG = 3x$ cm, $FG = x$ cm, $CD = DE$ and $DEFG$ is a rectangle.

(a) Find, in terms of x , the length of

(i) CD ,

(ii) EF .

(b) Show that the area enclosed by the wire is $[50\pi - 4x(5 - x)] \text{ cm}^2$.

(c) The area enclosed by the wire is $(50\pi - 21) \text{ cm}^2$. Form an equation in x and solve it.

(d) Is G the midpoint of AC ? Explain your answer.

(e) Find the minimum length of the wire. Give your answer correct to 3 significant figures.

20. The average speed of an aircraft in still air is 480km/h. On a particular flight, the aircraft flew 725km to its destination and the average wind speed is x km/h.

(a) Express the time taken by the aircraft to reach its destination in terms of x if it flew

(i) in the direction of the wind,

(ii) against the wind.

(b) The time taken to fly against the wind is 27 minutes longer than that taken to fly in the direction of the wind.

(i) Show that an equation in x is $9x^2 + 29\,000x - 2\,073\,600 = 0$.

(ii) Solve the equation and write down the speed of the wind.

Answers:

1. (a) $x = -8$ or $x = -6$
 (c) $x = 3\frac{1}{2}$ or $x = -4$
 (e) $x = \frac{1}{3}$ or $x = 3\frac{1}{2}$
 (g) $x = 2\frac{1}{3}$ or $x = -1\frac{1}{4}$
 (i) $x = 4$ or $x = -4$
2. (b) (i) $x = -3$ or $x = 2$
 (iii) $x = -3.4$ or $x = -0.6$
 (v) $x = -1.1$ or $x = 2.6$
3. (a) $b = 2$ or -2
 (c) $b = 9$ or -9
 (e) $b = 11$ or -17
4. (a) $x = -1$ or $x = -4$
 (c) $x = -0.697$ or $x = -4.30$
 (e) $x = 0.525$ or $x = -9.52$
 (g) $x = -0.562$ or $x = 3.56$
5. (a) $x = -3$ or $x = -4$
 (c) $x = 4.73$ or $x = 1.27$
 (e) $x = -0.573$ or $x = 2.91$
 (g) $x = -2.09$ or $x = 0.838$
6. (a) $x = 9$ or $x = -9$
 (c) $x = -1$ or $x = -8$
 (e) $x = 2.32$ or $x = -10.3$
 (g) $x = 0$ or $x = 3$
7. (a) $x + 4$ (b) $x = 5$ or $x = -9$ (c) 14
8. (a) $2y$ (b) $y = 3$ or $y = 0$ (c) 36
9. (a) $x = 6.83$ or $x = -7.33$ (b) $AB = 18.7\text{cm}$, $BE = 4.83\text{ cm}$
10. (a) $20x^2 + 2x - 121 = 0$ (b) $x = 2.41$ or $x = -2.51$
 (c) 242 cm^2
11. (a) $x = 3$ or $x = 7$ (b) $x = 1\frac{1}{2}$ or $x = -6$
 (c) $x = 1\frac{3}{4}$ or $x = 4$ (d) $x = \frac{3}{5}$ or $x = -1\frac{1}{2}$
 (e) $x = 6$ or $x = -6$ (f) $x = -1\frac{1}{2}$ or $x = 2$
 (g) $x = 1\frac{1}{5}$ or $x = 1\frac{2}{5}$ (h) $x = 3$ or $x = -8$
 (i) $x = 1\frac{1}{4}$ or $x = -2\frac{3}{4}$
12. (a) (ii) $x = -2.3$ or $x = 1.5$ (b)(ii) $x = -2.5$ or $x = 4$
13. (a) $x = 4.32$ or $x = -8.32$ (b) $x = 15.9$ or $x = -1.89$
 (c) $x = 0.541$ or $x = -5.54$ (d) $x = 6.37$ or $x = 0.628$
 (e) $x = 8.27$ or $x = 0.725$ (f) $x = 0.772$ or $x = -7.77$
 (g) $x = -0.889$ or $x = -28.1$ (h) $x = 7.87$ or $x = 0.127$
14. (a) $x = -0.268$ or $x = -3.73$ (b) $x = 8.41$ or $x = 0.595$
 (c) $x = 1.58$ or $x = -1.83$ (d) $x = -6.62$ or $x = 2.12$
 (e) $x = 5.30$ or $x = 1.70$ (f) $x = -0.842$ or $x = 14.8$

- (g) $x = -1.42$ or $x = -1.74$
15. (a) $x = -2$ or $x = -4$
- (c) $x = -1$ or $x = 2\frac{2}{7}$
- (e) $x = 1\frac{2}{3}$ or $x = -\frac{2}{5}$
- (g) $x = 11.4$ or $x = 2.64$
16. (a)(i) $x = 2^{\frac{2}{x}}$ (ii) $2^{\frac{4}{3}}$
17. (a) $a = -10, b = 24$
18. (a)(ii) $x = 14.6$ or $x = -0.137$
- (b)(i) $12\,942\text{ cm}^3$ (ii) 3367 cm^2
19. (a)(i) $x\text{ cm}$ (ii) $(20 - 4x)\text{ cm}$
- (c) $x = 3.5$ or $x = 1.5$ (d) No.
- (e) 54.4 cm
20. (a)(i) $\left(\frac{725}{480+x}\right)h$ (ii) $\left(\frac{725}{480-x}\right)h$
- (b)(ii) $x = 70.0$ or $x = -3292.2$, speed = 70km/h .